

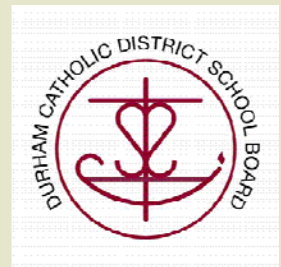
# Continuum Based Math

**Completed by:** Durham Catholic DSB

Project Team: **Mansel Legacy,**  
mansel.legacy@dcdsb.ca

**Mike Jacobs,**  
michael.jacobs@dcdsb.ca

MISA Lead: Paul Podesta,  
paul.podesta@dcdsb.ca





collected video evidence from students in our Board to highlight some common misconceptions. Here are some:

### **Kindergarten Counting**

The two students you see here are JK students (filmed in June). Notice how the boy in the first clip does not trust the count. Initially, this may be because he is so fixated with getting the blocks perfectly aligned that he *forgets* the count. Later though we see that he also doesn't touch each object carefully and as a result miscounts the collection. He thus gives three different answers (12, 13 and 14) for the same collection: no wonder he doesn't trust the count. As a result when two more blocks are out into the collection he does not count on, likely because he doesn't know what number to count on from!

In the second clip, notice how the girl is a lot more careful in touching and moving each of the blocks and how this technique results in her trusting the count. This in turn allows her to count on efficiently.

After seeing these clips, the teachers agreed the importance of making sure that their students do trust the count in getting them ready for future math understanding such as adding and subtracting. We also pointed out to the high school teachers that we have seen students in the Grade 9 Essentials class who still do not trust the count.

### **Decimals in Grade 7 (Quantity)**

We perhaps need to emphasise here that the responses given by the two students are not rehearsed or scripted. Also, their teacher considers them to be level 4 students. We know that students have great difficulty understanding decimals and this diagnostic gives great insight as to why: students don't truly understand the *quantity* of the decimals. That is to say they do not truly understand the place values of tenths, hundredths and thousandths. One student does know the place value *labels* of each column after the decimal point but does not know that 3.75 is larger than 3.6. Her misconception (a common one) is that the more decimal digits you have, the smaller the number (since, in their view, thousandths are smaller than hundredths which are smaller than tenths). The other student sees the decimal point as separating 'big' whole numbers from 'small' whole numbers and so reads 3.521 as "three point five hundred and twenty-one". This misconception is so solid that when she sees 6.125 in a money context she sees it as 'six dollars and one hundred twenty five cents' or in other words, \$7.25! We linked this to the high school teachers by getting them to consider this question:

*Which of the following lines is the steepest?*

- $y=2x + 4$
- $y=2.5x - 2$
- $y=2.25x + 3$

Teachers were able to see that if students have the same misconception as the second girl, they would choose  $y=2.25x + 3$ . They might understand the concepts of slope and y-intercept perfectly but make a mistake due to their misconceptions of quantity.

### **Arrays in Grade 2 (Operations)**

As highlighted by PRIME and FSIM, arrays are a useful way to move students on from repeated addition to understanding the operation of multiplication. In this clip from a Grade 2 class, we see students constructing their own arrays from a given number of counters (initially 18 then later 20). For each array, the students had to write a repeated addition sentence, and if they could, a multiplication sentence.

What we found interesting from this video was that whilst students were comfortable writing one repeated addition sentence for an array (and even writing the multiplication sentence too) they had great difficulty seeing the 'other way'. That is, for a 6 by 3 array, they could write  $3+3+3+3+3+3$  but *not*  $6+6+6$ . This is maybe not that surprising when one considers how the girls constructed the array: they added 3 dinosaurs at a time so this is the dimension of the array that they focus on. We as teachers (with all our worldly experience of working out the areas of rectangles) can 'see' both dimensions (i.e. how to count by rows and columns) but our students do not yet have that experience.

After seeing these clips, teachers realised the need for not just using arrays to link addition to multiplication but to explicitly draw the students' attention to how each array can be counted by either rows or columns. They could also see that memorising multiplication tables is not the same as learning about the operation of multiplication.

### **Addition and Subtraction Strategies in Grade 4 (Computations)**

In this video of a Grade 4 class in June, we can see the types of strategies used by a variety of students to tackle five questions. Each of these questions was written horizontally and the students were told that they could use any method they wished to get the answer. Notice how some students have used non-standard approaches and show a lot of good number sense in their explanations of how they got their answers. On the other hand, the student who chooses to use the traditional algorithm (even for  $1000-2$ ) shows very little number sense as well as performing mistakes in the required steps. We know that in the past, we as teachers ourselves might have re-taught the *steps* of the algorithm to this student but now we can see that there are other things which must be addressed first. For example, can this student *count* backward? If so, she could have just counted backward from 1000. Does this student understand the *quantity* or place value of the numbers? If so, maybe she could have done  $25-7$  by doing  $25-5$  to get 20 and then 20 subtract the remaining 2 to get 18 (instead of incorrectly using the algorithm).

It was really interesting showing teachers this clip. Many teachers themselves were drilled with traditional algorithms and were thus of the opinion that these algorithms should be taught as soon as possible to *their* students. Watching how these students responded though, made it easier for us to convince teachers that if algorithms are introduced too early, they are actually detrimental to a student's number sense. In a similar way, we were able show high school teachers that just teaching the steps for, say, an equation or using the Pythagorean Rule, will not necessarily lead to students being successful. What is more important is to get students to understand the processes involved or even use different methods according to the question.

### *Using the Tracking Charts*

The tracking charts proved to be very helpful in getting teachers to think further about having to change their practice. When they filled in the results from their diagnostics they

could see clearly where their students were having difficulties and where they were not. Initially some teachers were shocked to see how many gaps their students had. The key was to getting the teachers to, as one kindergarten teacher said, "... accept that this is where my students are. Now I know this, I can do something about it." As teachers could see clearly from the diagnostic what their students' misconceptions were, they were in agreement that, irrespective of what grade or phase they were in, they *wanted* to do something about it.

### *Teacher Reaction*

On the whole, teachers have been most positive about the training they received. Once they could see where their students had misconceptions they were keen to find out *why* these misconceptions arose. Some questioned how their teaching style might be reflective of how they were taught. Some questioned their own mathematical understanding. Some even questioned how their math teaching differed to their teaching in other subject areas. As one Grade 5 teacher said, "For years I have been so comfortable with the idea of giving students different books for reading according to their reading level. For Math, however, I gave all kids ***the same questions from the same page from the same book each day***. It didn't feel right but I never stopped to think why".

### *Textbooks, Curricula and Other Documents*

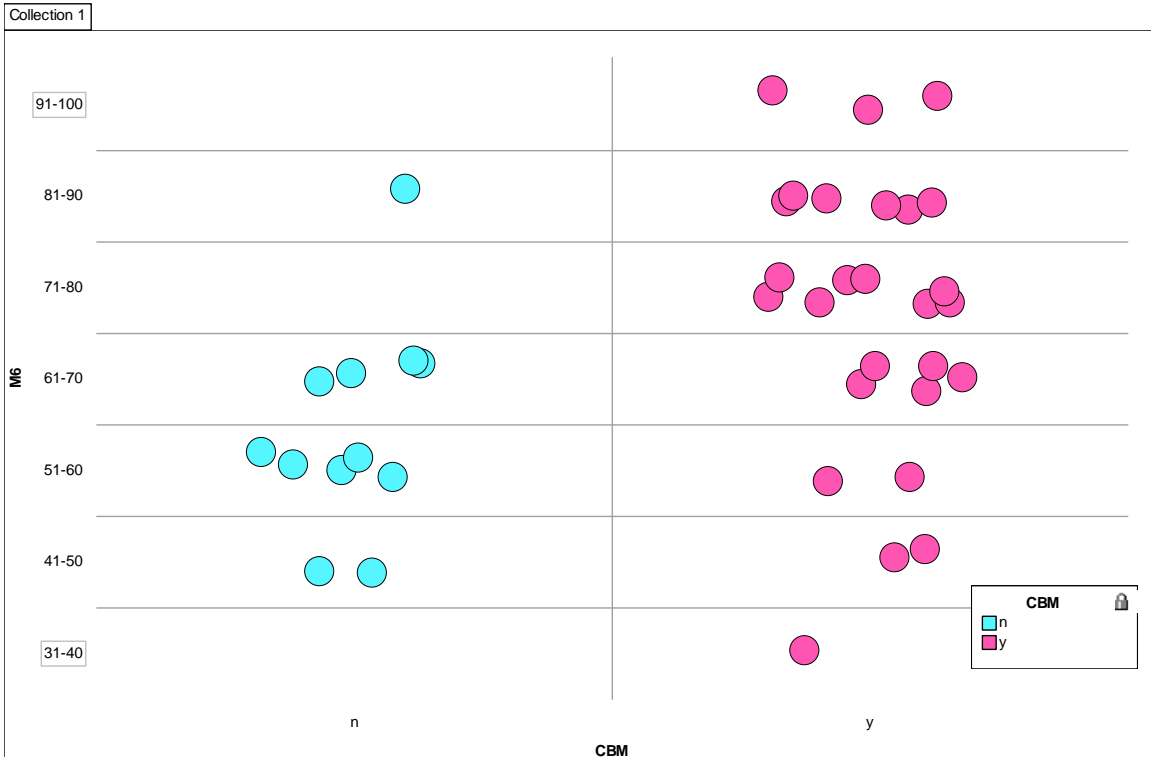
As a result, teachers had to critically examine the materials they were using to teach their students. In most cases, this was exclusively the textbook. We noted that many teachers were either not referring to the curriculum document or, if they were, referring to the old document. During our sessions we got teachers to firstly look at their curriculum to see what had to be covered and then look at the textbook. What they discovered shocked them: that there was a lot of material (that they had been spending a lot of class time on) that was *not* in the curriculum. We even had a number of teachers who, after seeing the curriculum, said "If this is *all* I've got to teach, then I can manage this". In other words, our training got them to use the curriculum to *free up teaching time for them!*

There then followed the question, what other resources can I use? At this point, we were able to get teachers to delve into such resources as the Guides to Effective Instruction, the TIPS4RM documents, Van de Walle, PRIME kits, FSiM books, various software (Geometer's Sketchpad, National Library of Virtual Manipulatives etc) as well as exploring the various manipulatives they already had in their classrooms.

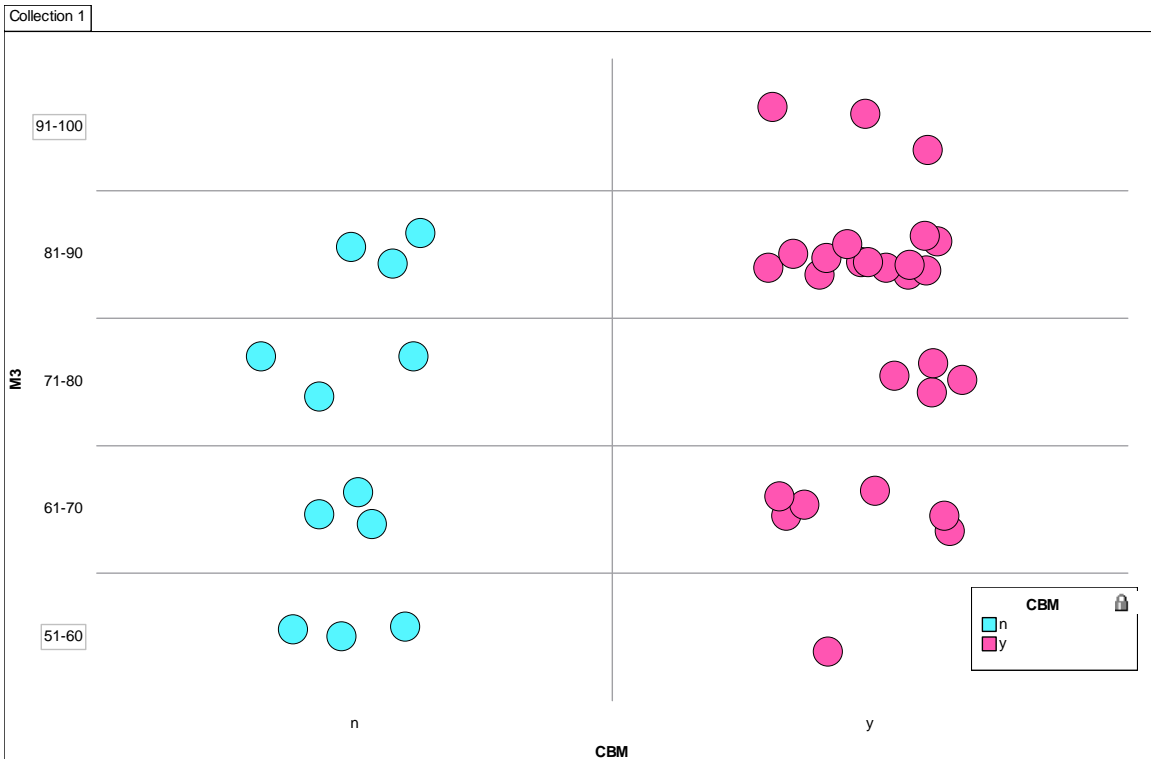
Teachers also began asking themselves (and us) questions about their evaluations and in some cases were surprised to see that what they were evaluating was not on the curriculum.

### *Preliminary Results of the Effects of CBM on Classroom Achievement*

It is difficult to measure the immediate effects CBM has had on EQAO results but the following graphs are certainly worth looking at. The first graph shows the percentage of students achieving the Provincial standard in Grade 6 EQAO Math and if they have received CBM training or not; *each circle represents one school*. The second is similar but is for Grade 3 EQAO Math. In both cases, schools that have been CBM trained were far more likely to have had students that reached the provincial standard than schools who hadn't been CBM trained.

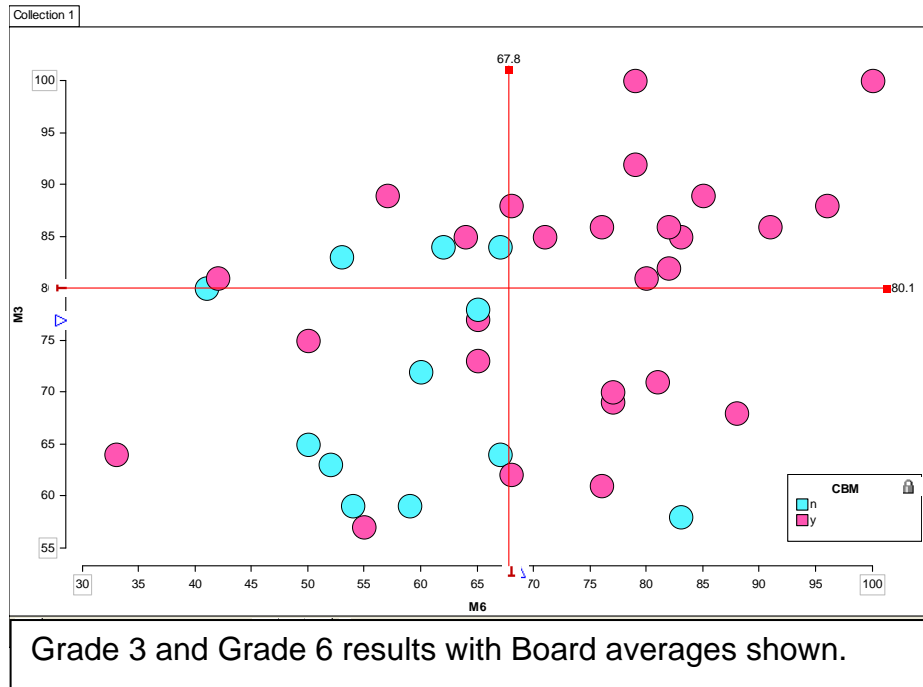


Percentage of students at Provincial Standard in Grade 6 EQAO Math.



Percentage of students at Provincial Standard in Grade 3 EQAO Math.

In the graph below, it can be seen that the only schools that performed above the Board average in both Grade 3 and Grade 6 EQAO Math were the schools that had received CBM training. Schools that had not received CBM training made a disproportionate amount of the schools that performed below the Board average for Grade 3 and Grade 6 Math.



### What Now?

So now we are in the position of having trained half the teachers in our Board and are now in the process of training all the remaining teachers. For those schools already trained it is vital that we keep the momentum going so we have arranged divisional meetings in these schools. Here, we meet with two or three teachers at a time and, under the CBM lens and using the data available from the tracking sheets, will either plan a unit, a lesson, or discuss some students' work. These meetings have so far been very productive.

Our Board is now planning to include the results of the diagnostics as a mandatory part of the assessment framework. This way, principals will be able to see if there are any discrepancies between CBM data, report card data and EQAO data.

We also need to do further analysis to see which particular concepts students are finding tricky across our Board and then further explore different activities that teachers might use to move students on, in particular activities that involve technology (clickers, interactive white boards).



## Phase 1 Diagnostic Task - Interview # 1 - Counting

**PURPOSE:**

Counting columns #'s - 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13

**MATERIALS:**

General purpose counters i.e., two-coloured counters, pattern blocks, blocks, .....etc.....

**INSTRUCTIONS:**

This is a one-on-one teacher/student interview. It should be completed in some privacy, away from other students. It is important that the script be followed in a precise manner, with little or no teacher help. Teachers may provide some clarification, but should not deviate from the overall goal of each task. Students should demonstrate a **'mastery'** of each task. ***Partially correct answers should not be marked as correct.***

<b>Teacher Instructions &amp; General Question for Student</b>	<b>Teacher 'Look Fors' Students should demonstrate <u>complete</u> understanding</b>	<b>Diagnostic Task – Chart #</b>
Put out 7-11 counters on the table before the student arrives (do not use 10) (do not have additional counters in sight) 1. <b>ASK:</b> “can you tell me how many are here?” (be sure not to say ‘count’ 2. <b>ASK:</b> “so how many are here?” (after the count is complete)	1. does the student count to find the total (as opposed to just guessing) 2. does the student count to find the correct amount 3. does the student count each object only once 4. does the student say the correct amount when asked	If correct check column #1
		If correct check column #2
Re-arrange the counters into a straight line Move one of the middle counters slightly out of line  1. <b>ASK:</b> “can you count again, starting from this counter? (pointing to the counter out-of-line)” 2. <b>ASK:</b> “so how many are here?” (after the count is complete)	1. does the student count all the counters 2. does the student get the same result as previous	If correct check column #3
With the counters still in a straight line.....point to the first counter in line  1. <b>ASK:</b> “how much is this counter?” .....point to the next counter in line 2. <b>ASK:</b> how much is this counter?”	1. does the student say that each counter is one	If correct check column #4

<p>.....point to the last counter in the line</p> <p>3. <b>ASK:</b> how much is this counter?"</p>		
<b>Teacher Instructions &amp; General Question for Student</b>	<b>Teacher ‘Look Fors’ Students should demonstrate <u>complete</u> understanding</b>	<b>Diagnostic Task – Chart #</b>
<p>Re-arrange the counters into a random order again</p> <p>1. <b>ASK:</b> “can you tell me how many are here</p> <p>2. Re-arrange and ask same question again (have student move the counters)</p> <p>3. <b>ASK:</b> “how do you know there is that many?"</p>	<p>1. does the student give the answer to the question immediately, without counting again</p> <p>2. is the student able to explain that there is the same amount because:</p> <ul style="list-style-type: none"> <li>• they already counted the objects</li> <li>• that none were added or taken away</li> <li>• that mixing (moving) the objects doesn’t change the count</li> </ul>	<p>If correct check column #5</p>
<p>1. <b>ASK:</b> can you start counting out loud and keep going until I say stop</p>	<p>1. is the student able to count the sequence of ‘11-19’ correctly</p> <p>2. is the student able to count up to 31 in sequence with <b>no errors</b></p>	<p>If correct check column #6</p>
		<p>If correct check column #7</p>
<h2>PHASE 2</h2>		
<p>1. <b>ASK:</b> can you start at 31 and count backwards until I say stop</p>	<p>1. is the student able to count backward from 31-0 in sequence with <b>no errors</b></p>	<p>If correct check column #10</p>
<p>1. <b>ASK:</b> can you start counting out loud and keep going until I say stop - start at 35</p> <p>2. <b>ASK:</b> can you start counting out loud and keep going until I say stop - start at 61</p> <p>3. <b>ASK:</b> can you start counting out loud and keep going until I say stop - start at 49</p> <p>4. <b>ASK:</b> can you start counting out loud and keep going until I say stop - start at 87 ..... 100</p> <p>5. <b>ASK:</b> can you start counting out loud and keep going until I say stop - start at 100.....end at 120</p>	<p>1. is the student able to count the sequence of ‘0-9’ through the decades correctly up to 100</p> <p>2. is the student able to count over 100 knowing that the sequence of numbers repeat (i.e., ‘0-10’, ‘11-19’ sequence)</p>	<p>If correct check column #11</p>
		<p>If correct check column #12</p>



