# Course Profile

## Principles of Mathematics

Grade 9
Academic

**!** *for teachers by teachers*

Course Profiles are professional development materials designed to help teachers implement the new Grade 9 secondary school curriculum. These materials were created by writing partnerships of school boards and subject associations. The development of these resources was funded by the Ontario Ministry of Education and Training. This document reflects the views of the developers and not necessarily those of the Ministry. Permission is given to reproduce these materials for any purpose except profit. Teachers are encouraged to amend, revise, edit, cut, paste, and otherwise adapt this material for educational purposes.

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# Unit 1: Constructing Graphical Models Through Investigation
**Time: 35 hours**

## Unit Description

Students will gather, analyze, manipulate, and display data from primary and secondary sources to model and communicate results about both linear and non-linear situations. Many contextual problems will be studied to ensure that students gain depth of understanding through meeting the same specific expectations in different contexts. Students will conduct investigations to verify or refute their own conjectures, using lines or curves of best fit, tables and pattern descriptions. They will communicate their findings and describe trends. A rich contextual foundation for subsequent algebraic studies will be built in this unit. Several different types of technologies will be introduced for gathering, analyzing and displaying data.

## Strands and Expectations

Some specific expectations from the Number Sense and Algebra, and Measurement and Geometry Strands have been combined with overall expectations REV.01, REV.02, REV.03 from the Relationships Strand. Weaving together the expectations of the strands in this way will help students make connections.

**Relationships Strand Specific Expectations:** RE1.01,02,03,04,05,06,07; RE2.01,02,03,04,05,06; RE3.01,02,03,04
**Number Sense and Algebra Specific Expectations:** NA 1.01, 03, 04; NA2.01,02,03, 04, 05, 06; NA 3.06; NA 4.01, 02, 03
**Measurement and Geometry Specific Expectations:** MG1.01,04; MG 2.01, 02, 03, 04

## Activity Titles

What follows is a suggested sequence, with timing, for teaching Unit 1. This profile develops only the activities that depart from traditional pencil and paper skill development. These activities are designed to help students make sense of mathematics by working through concrete experiences to develop their understanding of various mathematics concepts. The need for remediation and further development of skills will arise from the activities.* **Up to 960 of the 2100 minutes** have been allotted for work, as needed, on Cartesian graphing, integers, percents, exponents, basic algebra, graphing calculator and spreadsheet skills, to name a few. As the new elementary curriculum becomes fully implemented, the use of this time will change.

| | | |
|---|---|---|
| Activity 1: | What's My Style? Gather, Organize & Display Learning Styles Data (1 variable) | 75 minutes |
| Activity 2: | What's Our Class Profile? Gather, Organize & Display 1 Variable Data | 75 minutes |
| Activity 3: | Is There a Relationship Here? Searching for Two Variable Relationships | 150 minutes |
| Activity 4: | What Type of Relationship Is This? Interpret and Analyze Two Variable Data | 75 minutes |

*Time for: activity completion, other activities that address the same expectations, building graphing calculator skills, other **Skill Building**, quizzing (distribute as needed throughout activities) **200 minutes**

| | | |
|---|---|---|
| Activity 5: | A Cagey Problem! Discovering Linear & Quadratic Relationships between Geometric Measures | 90 minutes |
| Activity 6: | A Design Problem. Factors Affecting Steepness of a Linear Graph | 150 minutes |
| Activity 7: | Fold It! Discovering an Exponential Relationship Practising and Extending Exponent Skills | 75 minutes |

*Practise and extend **exponent skills** **225 minutes**

| | | |
|---|---|---|
| Activity 8: | Walk This Way! Describing Constant & Non-Constant Speeds in Distance/Time Relationships | 75 minutes |
| Activity 9: | Tell Me a Story: Interpreting and Analyzing Graphs in Contexts | 75 minutes |
| Activity 10: | What Does It Mean? Steepness as Rate of Change | 75 minutes |

*Time for: activity completion, other activities that address the same expectations, building graphing calculator skills, other **skill building**, quizzing (distribute as needed throughout activities) **160 minutes**
*Sample Assessment activities (practise performance tasks form EQAO, OAME, NCTM) or further **skill development** **150 minutes**
*Time to **Consolidate Skills** **225 minutes**

| | | |
|---|---|---|
| Activity 11: | Testing Roofs: A Summative Assessment Activity | 150 minutes |
| Activity 12: | Testing Skills: Sample Questions for a Summative Pencil and Paper Test | 75 minutes |

## Prior Knowledge Required

See Appendix.

## Unit Planning Notes

- The first activity is intended to help the teacher discover the range of learning styles in the class, and to help students better understand themselves as learners. Teachers might use this information to balance and adjust the types of learning and assessment activities that they use.
- Introduce linear and non-linear relationships using concrete materials, and use words, units, and representative letters from a wide variety of contexts, with no $x$'s and $y$'s until Unit 2.
- Practise using spreadsheets to record data and draw graphs, graphing calculators to enter data into lists and display graphs using the appropriate lists, Calculator Based Laboratory (CBL) and Calculator Based Range (CBR) equipment to gather and display data.

**Note:**
Only activity 8 requires the use of graphing calculators and CBL or CBR equipment. It is expected that this activity will be used some time after September, giving teachers time to become comfortable with the introductory applications of technology that this activity requires. Should it happen that graphing technology is not available to the teacher during Unit 1, it is recommended that the use of activities 8 & 9 be postponed until later in the course. Together, these activities present a powerful way for students to develop a concrete understanding of rate of change and spacial sense involving interpretation of

graphs.
There are many more opportunities for the use of technology than those suggested in this profile, and it is possible to find many activities that require no technology. This profile shows many more sample activities requiring no technology than requiring it. As teachers get more of the equipment in their schools and become more comfortable with its use, they may gather more activities that benefit from the use of technology.

- Gather examples of other activities, in different contexts that address the same expectations as each activity outlined. You may wish to use these as follow-ups, warmups, or substitutes. New textbooks and publications from OAME/OMCA and others will contain appropriate examples.
- Develop a plan and gather materials appropriate to diagnose and develop prior learning skills as they are identified throughout the unit. Some students will need remedial work to perform at the expected standard. **16 hours out of 35 hours** have been earmarked in Unit 1 for skill development. It is to be used throughout the unit as the need for skills arises within the contexts of the activities.
- Involve students whose prior learning skills are not in need of remediation, in extending questions and alternate extending activities.
- The relative need for remedial vs extending experiences may vary according to the group of students in a class.
- The concrete experiences presented in unit 1 are important preparation for the abstract algebraic treatment of linear relations that will occur in unit 2. Both units give students opportunities to develop higher level thinking and communication skills, an expectation prominent in the curriculum policy document.

## Teaching/Learning Strategies

There are many types of skills required of students as they engage in the activities of this Unit. In the teacher facilitation section of each activity, skills of the following types have been identified, **in bold**, as they are needed: **numeric skills, measurement skills, communication skills, analytic skills, use of technology skills,** and **collaborative skills**. A teacher may find that direct instruction is needed for any of the skills identified. It is intended that repeated, short, direct instruction in these skills, within a wide variety of contexts, will improve student skills in an incremental and natural way.

Teachers will be working diagnostically with students to determine which students do and which do not need remedial support on items identified in the Prior Learning section of each activity. Direct teaching of necessary skills for one group can happen at the same time that extending and enriching activities are used by another group of students. Flexibility of timing and structure are needed so that all students are engaged in meaningful tasks. Time has been allotted for this in the *asterisked time periods.

## Assessment/Evaluation Techniques

When students do open-ended, multi-dimensional work that requires them to perform in a situation which calls for mathematics, it is not useful to score their work on the basis of right or wrong, alone. Inviting students to show what they know and explain their reasoning means assessing a piece of work, or an entire performance, as opposed to a correct or incorrect solution. Teachers will need to look at the strengths and weaknesses of the whole piece of work or an entire performance, as it pertains to the specific expectations conveyed in the purpose of the task. Work can be scored holistically, with consistent standards, using a rubric. Rubrics are required when there is a range of student responses possible and

when there is a need for teachers to be much more precise about criteria for assessment. Several examples of assessment activities and their scoring rubrics have been included in this Profile. Rubrics will be the most effective means of measuring student performance on the Thinking/Inquiry/Problem Solving and Communication and Applications in unfamiliar settings categories of the Achievement Chart.

Most traditional pencil and paper tests do not offer students opportunities to demonstrate Level 4 performances. This Profile includes sample questions for pencil and paper tests that do allow students to demonstrate Level 4 work.

## Resources


Heid, M., Algebra in a Technological World. Addenda Series. 1995 NCTM
Asp, G. et. al., Graphic Algebra : Explorations with a Graphing Calculator. Key Curriculum Press
Meridian Creative Group, CBL Explorations in Algebra
Murdock, J. and E. Kamischke, Advanced Algebra Through Data Exploration. Key Curriculum Press
MCTM/SIMMS Integrated Mathematics A Modeling Approach Using Technology. Simon & Schuster Custom Publishing, 401 Linfield Hall, Bozeman, MT 59717-2810
Specht, Jim, More Than Graphs: Activities for TI Graphics Calculators. 1996 Key Curriculum Press
Coxford, A. et. al., Contemporary Mathematics in Context. 1997 Everyday Learning Corp., P.O. Box 812960, Chicago, Il 60681 (ISBN 1-57039-475-X)
Texas Instruments Real-world Math with the CBL System; 25 Activities Using the CBL and TI-82
Texas Instruments Explorations-Modeling Motions: High School Math Activities with the CBR, 1997
MARS - Mathematics Assessment Resource Service.**http://www.educ.msu.edu/mars**


## Appendix - Prior Mathematics Knowledge That Students Bring to Unit 1

### Patterning and Algebra

- find patterns and describe them using words and algebraic expressions
- write an algebraic expression for the *n*th term in a numeric sequence
- complete a table of values and write words to explain the pattern (from grade 7)
- use variables to write equations and algebraic expressions from patterns and complex statements

### Data Management

- collect primary data using both a whole population (census) and a sample of classmates
- assess bias in data collection methods
- manipulate and present data using spreadsheets or search databases for information and use the quantitative data to solve problems
- construct frequency tables, stem-and-leaf plots, line graphs, comparative bar graphs, circle graphs, and histograms, with and without the use of technology, and use information to solve problems (e.g. extrapolate ..., predict ...)

- read and report information about data presented on the graphs listed above
- understand the difference between a bar graph and a histogram
- know that a pattern on a graph may indicate a trend
- from grade 8 geography: construct a variety of graphs, charts, diagrams, and models to organize information (e.g. graphs that demonstrate correlations between two population characteristics, such as literacy and birth rates)
- understand and apply the concept of the best measure of central tendency and determine the effect on a measure of central tendency of adding or removing a value
- make inferences and convincing arguments that are based on data analysis

## Number Sense and Numeration

- compare and order fractions, decimals and integers
- arithmetic operations including order of operations involving fractions, decimals and integers
- solve and explain multi-step problems involving simple fractions, decimals, integers, percents, ratios and unit rates
- express repeated multiplication as powers
- express whole numbers in expanded form using powers and scientific notation

# Unit 1 - Constructing Graphical Models Through Investigation
## Activity 1 - What's My Style?
## Time: 75 minutes

### Description

In this introductory activity students will gather information about their own learning styles using a learning styles inventory (a website reference is provided) and organize their results using graphs. The data from this activity will be used for analysis in a number of activities which follow. In addition, information on students' learning styles will give mathematics teachers pedagogically useful information about their grade nine students.

### Strand(s) and Expectations

**Strands:** Relationships; Number Sense and Algebra
**Expectations:** This activity reviews expectations RE1.0, RE1.06 and NA1.03 in the context of one variable data in preparation for the work with two variable data that follows.

### Planning Notes

The activity described here makes use of the learning styles inventory "Assessing Your Learning Style", which can be found at <http://snow.utoronto.ca/Learn2/lstyle.htm> . This activity can be adapted for use with another learning styles inventory, if the teacher so chooses. Copies of the learning styles inventory are needed for each student in the class. A class list chart with columns to record each student's scores in each of the two learning styles categories is needed. A blank copy of this chart could be prepared on an acetate for use with the overhead projector, or in the form of a wall chart to be posted in class.

The student's scores would be compiled and computed by the student as follows, in this sample chart:

| Students | List A scores | **List B scores** |
|---|---|---|
| Andrew | 14 | 9 |
| Thansha | 6 | 12 |
| Iris | 7 | 15 |
| Class total scores | 27/60 | 36/63 |
| Class total scores as percentages | 45% | 57% |

A second chart to tally students' preferred learning styles should also be prepared ahead of time. A sample is provided.

| | Tally | Total Number of Students | Percentage of Students |
|---|---|---|---|
| stronger auditory | | | |
| stronger visual | | | |
| similar strengths | | | |
| class totals | | | |

The teacher should become familiar with the learning styles inventory ahead of time in preparation for any questions which may arise.

## Prior Knowledge Required

Percent calculations, construction of circle and bar graphs.

### Teaching/Learning Strategies

**Student Activity:** Students will complete the learning styles inventory and tally their scores for each of the different learning styles. These results will be copied onto the class list. Once the class results are compiled, the students will discuss ways to organize the learning styles data for the entire class (e.g. bar graph, circle graph) and together graph them.

**Teacher Facilitation:**

- Introduce learning styles and how information about learning styles can be used by students and their teacher.
- Introduce the activity and inform the class that this information will be used in subsequent activities. Also provide **numeric skills** instructions for students to evaluate and score their own learning styles inventory results and assign this task to the students. (5 minutes).
- Circulate around the classroom, assisting students in completing their learning styles inventories as needed (20 minutes).
- Compile the learning styles scores on the class list chart (10 minutes).
- Review/demonstrate **numeric skills** of percent calculation, and **graphical communication skills** of how to construct bar and circle graphs by constructing graphs of the entire class's results as follows: (20 minutes)

  1) find the class's total scores in each category

  2) calculate each total as a percentage of the maximum number of responses possible in each category

  3) tally the numbers of students whose learning style preferences lie in each of the three

categories (auditory - higher on list A, visual - higher on list B, tactile-kinesthetic - approximately the same on lists A and B)

4) calculate each score in 3) as a percentage of the entire class in preparation for constructing a circle graph

5) point out the difference between the class total scores as percentages and the most prominent styles as percentages. Relate these to mean and mode and point out that in this case mean takes into account all of the data whereas mode takes into account only part of the data

6) construct a bar graph to show the relationship between the response rates (as percentages) for the list A and B items within the class. Then construct a circle graph to show the distribution of students amongst the three learning styles categories

- Assign students the following task as part of their homework (5 minutes). Add whatever skills questions seem appropriate. Emphasize to students that they should demonstrate their **numeric, communication, and analytic skills** in completing their homework.

**Student Activity:**

1) Construct a bar graph to compare your response rates for the list A and B items and answer the following question: "How does your learning style profile fit that of the class as a whole?"

2) Form a hypothesis about the similarity of your own learning style profile with that of a family member or friend, and test your hypothesis by administering the learning styles inventory to that person so that the results can be compared.

## Assessment/Evaluation Techniques

None.

## Accommodations

The teacher should be prepared to assist individual students who have difficulty performing these tasks. Parts of the inventory may be completed as a whole class exercise to get the students started and individuals who have difficulty with the text may be grouped with other students for assistance. If a student has difficulty with a question it may be left blank.

## Resources

The teacher may wish to do some research on learning styles and how to incorporate this into their teaching. The website cited above is a good start. Another useful reference is Teaching and Learning Styles: Celebrating Differences, OSSTF, 1986.

## Activity 2 - What's our class profile?
## Time: 75 minutes

### Description

In this activity, students will continue to gather information about themselves and the class. Following a brief review of graphing, the students will graph one item from the class data and draw an inference about the class from this.

### Strand(s) and Expectations

**Strand:** Relationships
**Specific expectations:** The activity relates to expectations RE1.04, RE1.05, and RE1.06, in the context of one variable data, in preparation for the work with two variable data that follows.

### Planning Notes

Needed are copies of the Personal Traits Questionnaire found at the end of this activity, and a class list chart with ten columns, to record students' responses for each of the eight questions from the questionnaire. Part A of the questionnaire will be completed during activity 2 and part B, during activity 3. As in the first activity, this chart could be prepared on an acetate, or in the form of a wall chart. Prepare examples of a comparative bar graph, and a histogram, and calculations for measures of central tendency. The teacher should assist in making available the materials students will need to present their group graph to the class (e.g., chart paper, markers, overhead acetates, compasses, protractors, and calculators). Prepare a homework task where students will interpret information from various types of graphs.

### Prior Knowledge Required

Using mode, organizing and graphing data.

### Teaching/Learning Strategies

**Student Activity:** Discuss responses from the previous day's homework. Students will complete part A of the personal traits questionnaire and have their answers recorded on the class list by the teacher. Working in groups, students will graph class data and look for trends.

**Teacher Facilitation:**

- Lead the class discussion of the previous day's homework on learning styles, helping students with their **communication skills**. Conclude this by having students relate the different learning styles to specific behavior patterns that are familiar to them. (e.g., an auditory learner may prefer radio to TV) (10 minutes).
- Get the students started on part A of the personal traits questionnaire.
- Record students' answers for the personal traits questionnaire as soon as they have completed it (15 minutes).

- Provide a review of the **communication skills** used and construction of graphs and the **numeric skills** of calculation of mean, median, and mode from the raw data, and the **analytic skill** of choosing the appropriate use of each of these measures. (This should be a quick review of grade 8, not a lesson.) (10 minutes).

  Brainstorm ways in which to demonstrate **collaborative skills** of working effectively in a group. The following guidelines are suggested:

  1) Assign specific roles to each member of the group. For the current activity these could be coordinator, person who calculates, graphics designer, and writer. (Assign roles to special needs students first to encourage their participation)
  2) Each person contributes to their group's work.
  3) Listen carefully to what other group members say and ask questions when needed.
  4) Help and encourage other members of your group.
  5) Keep working until everyone in your group understands your results and can explain them fully.

- Organize the class into ten groups and assign each group the task of graphing the class results for one of the questionnaire items. Direct students to use **numeric skills** to calculate an appropriate measure of central tendency for their data, and to use **communication and analytic skills** to prepare a description of any trends or suggestions about the class indicated by their graph and to justify their conclusions.
- Circulate within the class and help individual students as needed. (15 minutes)
- Have the groups use their **communication skills** to present their completed graph to the class and share their conclusions. (15 minutes)
- Have each group use their **analytic skills** to complete a single evaluation of their teamwork.
- Introduce the homework. (5 minutes)

## Assessment/Evaluation Techniques

Observations of student's team work and organizational skills could be recorded at this time. Students could be asked to reflect on their group's work using the following questions.

Group Evaluation

1. What made your group effective as a team?
2. What could you do differently to work more effectively as a team in the future?
3. Assess your group's efforts under the following criteria. Place a checkmark in the appropriate space for each.

| | | **Rarely** | **Sometimes** | **Usually** | **Always** |
|---|---|---|---|---|---|
| a) | Every person contributed to | _____ | _____ | _____ | _____ |

| | | | | | |
|---|---|---|---|---|---|
| | the group's work. | | | | |
| b) | Every person listened carefully to other group members. | _____ | _____ | _____ | _____ |
| c) | We asked questions when needed. | _____ | _____ | _____ | _____ |
| d) | We helped and encouraged each other. | _____ | _____ | _____ | _____ |
| e) | We kept working until everybody in the group understood the results and could explain them fully. | | | | |

## Accommodations

Assist individuals as needed and provide examples that students can use as a reference. The inventory can be read to students and responses can be scribed.

## Resources

There are many examples of the different types of graphs in current textbooks, magazines, and newspapers for students to find and bring in.

## Appendix

## Personal Traits Questionnaire

Part A: Answer the following questions in the spaces provided.

1. What is your gender, male or female? _____

2. What is your age in months? Round down to the nearest whole month. _____

3. On average, how much time in hours do you spend watching TV per day? _____

4. How many children (including yourself) live at your home? _____

5. On average, how many hours to you sleep each night? _____

6. How much time in minutes, on average, did you spend on your homework each school night during your previous semester at school? _____

7. On average, how much time in minutes do you spend listening to music each day? _____

8. On average, how many minutes do you spend reading per week for your own enjoyment? _____

9. How much time in hours, on average, do you spend per week doing physical activities such as exercise, active games, sports, etc. Round off to the nearest half hour. _____

10. On average, how many hours do you spend per week doing chores or other household tasks? _____

Part B: You will complete this section with the help of a partner.

11. Hand span is the distance from the tip of your thumb to the tip of your baby finger when your hand is stretched out as far as possible. What is your span of your right hand in centimetres? _____

12. Forearm length is measured from the crease on the inside of your elbow to the crease on the inside of your wrist. What is length of your left forearm in centimetres? _____

13. Arm span is the distance from finger tips to finger tips when your arms are stretched out horizontally as far as possible. What is your arm span in centimetres? _____

14. What is the length of your right foot in centimetres? _____

15. Stride length is the distance from the tip of your toe on one foot to the tip of your toe on the other when you take a normal walking step. What is your stride length in centimetres? _____

16. What is your height in centimetres? _____

## Activity 3: Is There a Relationship Here? Searching for Two Variable Relationships
## Time: 150 minutes

### Description

This activity introduces the students to two variable relationships and the possibility that some sets of data will not yield a relationship. Students will use data collected in the previous activity and newly collected data to form an hypothesis, find relationships between variables, make scatter plots, choose appropriate axes and draw a line or curve of best fit. The graphing calculator projection panel can be used as a demonstration to display data and underline the need for decisions concerning scale.

**Strand:** Relationships
**Specific Expectations:** RE1.01, RE1.04, RE1.05, RE1.06, RE2.02, RE2.0, RE2.05

### Planning Notes

- The teacher will supply metre sticks, measuring tapes and the activity outline, "Is There a Relationship Here?" Each student should have a copy of the Personal Traits Questionnaire for the previous activity.
- Emphasis will be placed on students exploring and constructing relationships between sets of data. The data from Activities 1 and 2 should be displayed on a bulletin board or wall or organized using a spreadsheet and made available to the students. The class set of data must not identify individual students by name to avoid any discomfort that may stem from a discussion of physical characteristics.
- It is appropriate to introduce the notion that there are formal, mathematical ways to find a line of best fit. This would be an excellent springboard for discussion about criteria for such a line and the need to communicate these criteria clearly. Teachers may wish to use Fit-ness from the Harvard Balanced Assessment before or during this activity to introduce students to the criteria for choosing a line of best fit.
- The view screen for the graphing calculator is a useful device to illustrate the scatter plot of the data and generate discussion around the 'viewing window' using technology and by hand with the class. Teachers will instruct students on how to use the list, stat plot and window functions on a graphing calculator.

### Prior Knowledge Required

*Patterning and Algebra Grade 7 and 8:* Recognize patterns and use them to make predictions.

*Data Management and Probability Grade 7 and 8:* Manipulate and present data using spreadsheets; identify and describe trends in graphs, using informal language to identify growth, clustering, and simple attributes; know that a pattern on a graph may indicate a trend.

*Grade 8 Geography:* Constructs a variety of graphs, charts, diagrams and models to organize information (including correlations between two population characteristics).

## Teaching/Learning Strategies

**Student Activity:** Students will work in pairs to measure their hand spans, forearm lengths, arm spans, foot lengths, stride lengths and heights, to complete part B of the personal traits questionnaire. Students should record their data using the data base or wall chart started earlier.

**Teacher Facilitation:** The teacher will group the students in same gender pairs and distribute the materials for this measurement activity. While students are working, the teacher will circulate to assist students with the **measurement and collaborative skills** needed to complete the task.

**Student Activity:** Students will work with a partner to carry out the following, "Is there a Relationship Here?" Upon completion of the activity, students will present their findings to the class, reporting on questions 1, 3, and 5.

## Is There a Relationship Here?

A relationship between two sets of data can help you to make predictions that lead to informed decisions about important matters such as setting up a small business or the effectiveness of a new medicine. You will be looking for connections between two of the data sets that you have collected. For example, can foot size be used to estimate a person's height? Is the amount of time that you spend watching TV related to the amount of time that you spend on homework?

1. What question about the sets of data that have been collected would you like to answer? Write this question down and then form a hypothesis. The hypothesis is your best guess, it answers the question , "What do you think the relationship will be?"
2. Create a scatter plot of the relationship in #1. Answer these questions:
   a. What are your variables?
   b. On which axis will you place each variable?
   c. What scale will you use for each axis?
3. Does your scatter plot show a definite relationship between the data sets? If so, describe the relationship. Consider these questions:
   a. Does the scatter plot reveal an obvious pattern?
   b. Do the points seem to describe a straight line or a curve?
   c. Does the value of one variable increase or decrease as the value of the other increases?
4. Make a list of the criteria for drawing a line of best fit that were identified by the class. Using the list of criteria, draw a line or curve of best fit through the points on your scatter plot.
5. Describe the relationship between your data sets. Consider these questions:
   a. What are the variables that are represented in this relationship?
   b. Is this relationship linear or non linear?
   c. Does the value of one variable increase or decrease as the value of the other increases?
   d. Is the rate of increase or decrease low? high?
   e. Does the relationship support your hypothesis? If not, create a new hypothesis. Explain your reasoning.
   f. Can you think of an equation that would describe this data?
6. a. Plot your data using the list and stat plot functions of your graphing calculator.
   b. Set the window and explain your choices.
   c. Describe and explain differences between your hand scatter plot and the calculator's image.

**Teacher Facilitation:** The teacher will set the stage for this activity by describing the following scenario:

*Marketing analysts study data gathered from a population to predict social trends so that they can decide whether or not a product or service will be in demand in a certain area. It is often necessary to know if there is a relationship between two variables or not.*

Brainstorm with the whole class to develop **analysis skills**. Ask students to identify sets of data that may be related (e.g., relationship between hand span and foot size, amount of time spent watching TV and score for a specific learning style, arm span vs height). Choose one example, formulate a hypothesis and discuss how you would prove or disprove it using the class data. Discuss ways to represent the data visually. Review the **graphical communication skills** involved in plotting points on a grid to create a scatter plot. The class should also consider the decisions to be made about scale and whether or not it is appropriate to use a break in an axis. An arbitrary decision regarding the placement of variables on the axes must also be made by the students since dependence and independence will not be obvious at this stage.

The goal of this activity is to identify both linear and non linear patterns, and to realize that some sets of data are not related in an obvious way. The students' **communications and analysis skills** will dictate how much guidance is needed in formulating an hypothesis. The teacher will direct students as needed, to use a scatter plot.

During the activity, the teacher will circulate and prompt. When most students have worked on question 3, the teacher will lead a whole class discussion to develop further **analysis skills.** Students will be asked to list criteria for a line of best fit (e.g., distance from the points, number of points above and below the line, number of points on the line, whether or not the origin is on the line, ignoring outliers). The students should not draw a line of best fit before the class discussion that identifies the criteria for this task. Students can judge each others work to determine *the* line of best fit. Some students may extend their **communication skills** by writing an equation to describe their line, but this is not necessary at this point.

As students use their **communication skills** to present their findings, the teacher will ask probing questions as needed to ensure that the key concepts described in the activity description emerge, using **analysis skills** (e.g., How does the trend shown in your graph compare to your hypothesis?).

## Assessment/Evaluation Techniques

The teacher should make observations regarding the students' independence and teamwork skills using the rubric in Appendix 1. Observational assessment can also be used together data on students' knowledge, and problem solving skills. This would be the ideal time to provide formative assessment feedback to students regarding their communication skills using "Rubric for Assessing Student Presentations".

## Appendix

### Rubric for Assessing Student Presentations

| | | | | |
|---|---|---|---|---|
| Formulate a hypothesis associated with a relationship between two variables<br><br>RE 1.01 | • Identifies the variables, with help<br>• Formulates a hypotheses that does not describe the relationship | C Identifies the variables, with help<br>C Formulates a hypothesis that describes the relationship, with some help | C Identifies the variables<br>C Formulates a hypothesis that describes the relationship | C Identifies the variables<br>C Formulates with confidence a hypothesis that describes the relationship |
| Describe trends and relationships observed in data<br><br>RE 1.05 | C Makes correct inferences from data, with much teacher support<br>C Explanations of the differences between the inferences and the hypothesis unclear and incomplete<br>C Has difficulty discussing any relationship that exists in the data | C Makes correct inferences from data, with some prompting<br>C Explanations of the differences between the inferences and the hypothesis unclear<br>C Discusses only one relationship that exists in the data with minor errors | C Makes correct inferences from data with minor errors<br>C Explains the differences between the inferences and the data<br>C Discusses only one relationship that exists in the data | C Makes correct inferences from data<br>C Clearly explains the differences between the inferences and the hypothesis<br>C Discusses other relationships that might exist in the data |
| Communicate the findings of an experiment<br><br>RE 1.06 | C Communicates unclearly<br>C Uses little or no justification | C Communicates results with some inappropriate forms<br>C Uses faulty logic to justify conclusions | C Communicates clearly using appropriate forms<br>C Justifies relationship with respect to the class | C Communicates and justifies clearly and concisely<br>C Generalizes relationship beyond the context of the classroom |

## Activity 4: Interpreting and Analyzing Two Variable Data
## Time: 75 minutes

### Description

This activity builds on the students' ability to identify linear and non linear relationships using tables and graphs based on primary and secondary data. Data will be selected by the teacher to allow for the use of finite differences (particularly the first differences) in the classification of relationships as linear or non linear. Dependence and independence, and the four quadrants of the Cartesian plane will be introduced in context as needed.

**Strands:** Number Sense and Algebra, Relationships, Analytic Geometry
**Specific Expectations:** NA1.01, RE1.04, RE2.01B, RE2.02B, RE2.04B, RE2.06B, RE3.03B, AG1.01

### Planning Notes

A variety of data tables that represent linear (with positive and negative slopes) and non linear relationships must be prepared for this activity. Current (secondary) data could be gathered from the Internet (for example, at www.statcan.ca ), but keep in mind that this activity requires data with a readily apparent relationship between the variables. Data can be generated easily using simple formulas from science (distance vs time, speed vs time, volume of a gas vs temperature, voltage vs current, mass vs volume, conversion between Fahrenheit and Celsius temperature scales) and other everyday applications (loan balance, bank account balance, sales tax gas consumption, perimeter vs area, metric conversions).

The calculation of first differences will require the subtraction of $2^{nd}$ minus $1^{st}$, $3^{rd}$ minus $2^{nd}$, etc, values of the dependent variable. The subtraction of integers should be reviewed here to facilitate finding finite differences when negative integers are used. A number of the tables should contain data with negative values and/or a negative slope.

Linear and non linear graphs are needed for the second half of this activity. The graphs should include grid lines or dots and the scales must be easy to read.

The concepts of dependence and independence may be introduced when the students are choosing a variable to start constructing their tables of values. A class discussion about whether or not it matters which variable is chosen in order to read the value of the other variable, and when it matters in the design of an experiment could clarify these issues for the students. (e.g., If a person was traveling at 100 km/h on a highway, the distance traveled would depend on the time traveled.)

### Prior Learning

*Number Sense and Numeration Grade 8:* Add and subtract integers.

*Patterning and Algebra Grade 7 and 8:* Recognize patterns and use them to make predictions.

*Data Management and Probability Grade 7 and 8:* Identify and describe trends in graphs, using informal language to identify growth, clustering, and simple attributes; know that a pattern on a graph may indicate a trend.

### Teaching/Learning Strategies

**Student Activity:** Students will carry out the following activity in small, heterogeneous ability groups.

## Interpreting and Analyzing Two Variable Data

1. A stone is dropped off the top of a tall building. Two sets of data were recorded and are shown below.

i)

| Time (s) | Speed (m/s) | First Difference in the speed (new value - previous value) |
|---|---|---|
| 0 | 0 | <10-0 = |
| 1 | 10 | < |
| 2 | 20 | < |
| 3 | 30 | < |
| 4 | 40 | < |
| 5 | 50 | |

ii)

| Time (s) | Distance (m) | First Difference in the distance fallen |
|---|---|---|
| 0 | 0 | < |
| 1 | 5 | < |
| 2 | 20 | < |
| 3 | 45 | < |
| 4 | 80 | < |
| 5 | 125 | |

Using the entries in the first two columns of each table, graph each relationship on a separate grid. When choosing your axes, use time as the independent variable.

What is the speed of the stone and the distance fallen when the time is 2.5 seconds?

a) Explain your reasoning, including the assumptions you made. When would the speed of the stone be 80 m/s? Explain your reasoning.
b) When would the distance travelled by the stone be 245 m? Explain your reasoning.
c) Which relation is linear?
d) For the linear relation, form an equation that models the data.
e) Complete the third column for each table. Contrast the differences formed in the two tables.
f) How does the type of differences in the table relate to the shape of the graph?

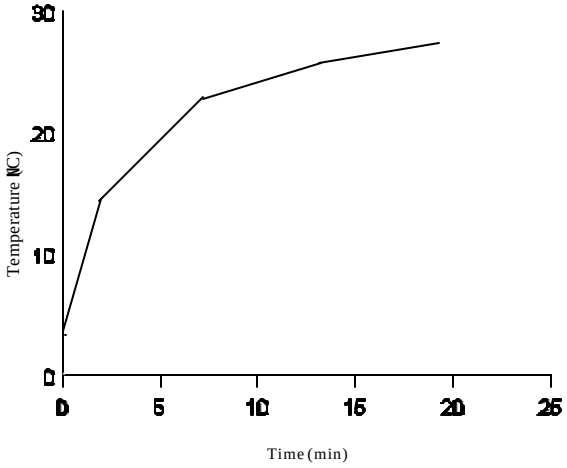
g) How does the constant first difference for table i) relate to the graph and to the equation for this relationship?

2. The air temperature on a cold, clear night is measured at the beginning of each hour.

| **Time (hours)** | **Temperature ( °C)** | **First Difference in the temperature** |
|---|---|---|
| 0 | -2 | |
| 1 | -5 | < |
| 2 | -8 | < |
| 3 | -11 | < |
| 4 | -14 | < |

- Calculate the first difference for the temperature.
- Do you think that this data describes a straight line or a curve? Explain your reasoning.
- Create a scatter plot of the data and draw a line or curve of best fit. Does the graph support your answer to part b?
- Find an equation to model the data.
- When would the temperature be -20 °C? Explain how you arrived at your answer.
- What would be the temperature if the time were 2.5 hours? Explain two different ways to answer this question.

3. A cold drink is left out in the sun on a warm day. The graph of its temperature vs time follows




a) Is this graph linear or non-linear?
b) What pattern do you think will exist in the first difference for this relationship? Explain your reasoning.
c) What decisions will you have to make before you choose points on the curve if you want to create a finite differences table?
d) Use the table to record the values of time and temperature for five points on the graph and calculate the first difference for temperature.

| Time (minutes) | Temperature (EC) | First Difference |
|---|---|---|

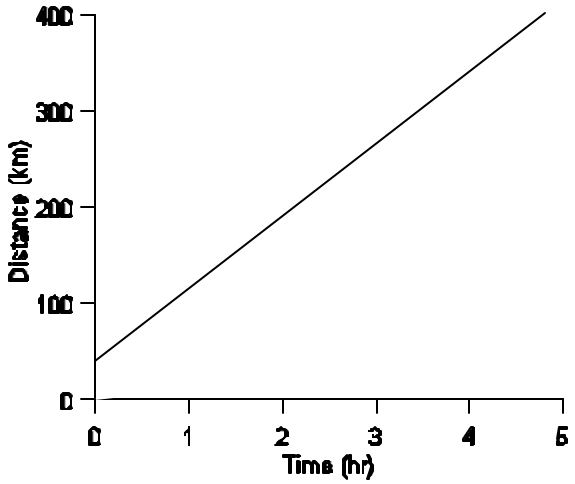
| | | |
|---|---|---|
| | | < |
| | | < |
| | | < |
| | | < |
| | | |

e) Do the first differences support your answer to part b? Explain why or why not.

4. Tara wishes to keep track of the number of kilometres that she travels on a basketball team road trip. She forgets to begin making observations until she is 50 km from home. She begins timing the trip (time equals zero hours) at the 50 km point. The graph below provides distance vs time data for Tara's trip.

a) Is this graph linear or non linear?

b) What pattern do you think will exist in the first differences for this relationship? Explain your reasoning.




c) Use the table below to record the values of time and distance for five points of your choosing on the graph and calculate the first differences for distance.

| Time (hours) | Distance (km) | First Difference |
|---|---|---|
| | | |
| | | < |
| | | < |
| | | < |
| | | < |
| | | |

d) Do the first differences support your answer to part b? Explain.
e) Estimate the amount of time that had elapsed before Tara began to record observations. Explain your reasoning and state any assumptions that you have made.

**Teacher Facilitation:** First differences (a subset of finite differences), dependence and independence, and discrete vs continuous functions are introduced in this activity. In question 2a, lead students to the understanding that they have been given a discrete sample of a continuous function. This means that points on the graph could be connected, and interpolation using a fractional time value would be valid. Averaging values of temperature would also be valid since the relationship is linear. Finish this activity with a class discussion that summarizes strategies for students to use when answering the questions including: patterns of first differences in linear and non-linear relationships (constant and not); averaging to interpolate; previous history and prediction for extrapolation. Allow for a brief discussion at the end of the class to introduce the homework assignment. This assignment should include questions that review essential integer **numeric skills**, as well as new **tabular and graphical communication skills.**

## Assessment/Evaluation Techniques

Collect and assess individual student work for accuracy of calculations, quality of communication and completeness. The following questions could be incorporated into a quiz in which the teacher also tests skills.

## Accommodations

Provide or post a chart outlining integer skills. Leave examples and guidelines, that were 'talked about', posted in the classroom.

## Sample Questions to Quiz Interpreting and Analyzing Data

**Name** _____

1. After the winter season, a swimming pool needs to be filled with water. A hose is left on for several days. The height of the water in the pool is given in the table below.

| Time, d, (days) | Height,h, (Metres) | First Difference |
|---|---|---|
| 0 | 1.5 | |
| 1 | 1.75 | |
| 2 | 2 | |
| 3 | 2.25 | |
| 4 | 2.5 | |
| 5 | 2.75 | |

a) Calculate the first differences for the height.
b) Do you think that this data describes a linear relation? explain your reasoning.
c) Create a scatter plot of the data and draw a line or curve of best fit. Does your graph support your answer to part b?
d) Find an equation to model the relation.
e) How many days did it take the hose to fill the pool to 2.1 m?
f) After 5.5 days, what is the water level in the pool?

2. Angela wonders if there is a relationship between the total volume of beverages she consumes and the amount of time spent watching television. She decides to keep a log for 5 days, then analyzes the data that she collects.

| Day | Beverages Consumed (ml) | Amount of TV Watched (hours) |
|---|---|---|
| 1 | 500 | 2 |
| 2 | 400 | 1 |
| 3 | 585 | 4 |
| 4 | 550 | 3 |
| 5 | 600 | 5 |

a) Create the first differences for the volume.
b) Does there seem to be a relationship between the volume of beverages consumed and the amount of time spent watching television. Is it linear? Explain your reasoning.

c) Draw a graph for the relation between time and volume.
d) Predict the amount of beverage she would consume if she watched TV for 3.5 hours.
e) Predict the number of hours she watched TV if she consumed 225 ml of beverage.

3. Angela reconsiders the variables and decides to examine the relationship between the number of hours that she watches television and the amount of orange juice that she drinks. Once again, she keeps a log for five days and then analyzes the data that she collects.
   a) Does there seem to be a correlation between the amount of orange juice that Angela drinks and the amount of time that she spends watching television? Justify your reasoning.
   b) Create a scatter plot for this data.
   c) Does your graph support your answer to part a)?

| Day | Orange Juice Consumed (ml) | Amount of T.V. Watched (hours) |
|---|---|---|
| 1 | 210 | 1 |
| 2 | 500 | 4 |
| 3 | 0 | 3 |
| 4 | 700 | 5 |
| 5 | 100 | 2 |

## Activity 5: A Cagey Problem - Searching for a Relationship Between Geometric Measures

**Time: 90 minutes**

### Description

Students continue to explore relationships between two variables by pursuing expectations from the Measurement and Geometry strand. Since students will be working with formulas, the data will be clean (no outliers and fitted exactly by a smooth curve). This unit also introduces the concept of optimization.

**Strands:** Relationships, Measurement and Geometry, Number Sense and Algebra

**Specific Expectations:** NA1.01, NA1.04, NA1.05, NA1.06, RE1.04, RE1.05, RE 1.06, RE 1.07, RE2.04, RE2.05, MG1.04

### Planning Notes

This activity might best be done in pairs so that students could share ideas and materials. However, each student should write out his or her own solution as outlined in the student activity description below.

### Prior Learning

*Patterning and Algebra Grade 7 and 8:* Recognize patterns and use them to make predictions; use the concept of variable to write equations.

*Data Management and Probability Grade 7 and 8:* Identify and describe trends in graphs, using informal language to identify growth, clustering, and simple attributes; know that a pattern on a graph may indicate a trend.

*Measurement Grade 7 and 8:* Apply the area formula to problem-solving situations.

### Teaching/Learning Strategies

**Student Activity:** Students will investigate the relationship between the perimeter and area of a rectangle. Student instructions:

### A Cagey Problem

You are working on the design of a home for a small animal. You have to determine the best dimensions of a rectangular base. The perimeter of the base is limited to 225 cm due to the high price of the fasteners that secure the top to the base. You are to investigate this problem and:

1. Form a hypothesis about the best dimensions for the cage.
2. Collect the data needed to support your hypothesis. Include well-labelled diagrams and computations.
3. Organize the data and predict the shape of the graph. Support your prediction.
4. Graph your data and describe how well it fits your prediction in step 3.
5. Prepare a recommendation for the best dimensions and include your supporting evidence. Ensure that tables and graphs have appropriate titles and units.

**Teacher Facilitation:** The teacher could provide graph paper and geo-boards, and lead a whole class brainstorming discussion, or visit individual groups, to guide **analysis skills** as students generate and critique criteria to use for the 'best' dimensions. The teacher may need to lead a short activity to review **use of technology skills** in showing how geoboards can be used. It is intended that the maximum area will be explored. Use **technology skills** through the use of spreadsheets or graphing calculators where possible to organize data for this activity. Students may have to be helped with **analysis skills** and led to the idea that as the length varies, the width is determined and the area changes. This discussion may be addressed with the class as a whole at the beginning, or with small groups during the activity. The teacher may have to remind individual students to consider the use of **numeric skills** of finite differences and judging reasonableness of answers to **analyze** their data in sequence for step 3.

## Assessment/Evaluation Techniques

Teachers may use the "Rubric for Assessing Student Presentations" from Activity3, with the category "collects and organizes data" added. (See Unit 4 Appendix, A Rubric for Observing Students, for ideas.)

Alternatively, a good strategy to help students see the kinds of things that they can do with a solution of an investigative problem, is to have students move into groups of 4 and receive photocopies of their classmate's work (remove names or use work from another class if there is more than one grade 9 class doing the same activity). Students then should sort the solutions into four piles (or 3 depending on the quality of the solutions the teacher has available). Each group should discuss the criteria they used to sort into piles and write these down so that they are able to discuss as a whole class. The whole class discussion should focus on building descriptors of how students sorted the piles. This could be done on the chalkboard. Through this discussion, students will start to understand the levels of performance on solving problems, and communicating methods and findings.

The teacher could then generate a rubric which includes four levels and incorporates the student suggestions. The teacher could use it to assess student work by circling the appropriate descriptors. The categories for assessment could include: clarity of communication of their hypothesis, predictions and recommendations; correctness of computations; applications of previously-learned skills. The purpose of this formative assessment is to provide feedback and suggestions for improvements.

## Activity 6: A Design Problem; Algebraic and Graphical Models of a Relationship
**Time: 150 minutes**

### Description

In this activity, students work with available, discrete, clean data gathered through explorations of volume, base area and height of buildings. Through construction of both algebraic and graphical models, the students investigate concepts such as steepness of graphs, appropriateness of scale and the creation and rearrangement of formulas. Extensions allow for further work on optimization problems.

**Strands:** Number Sense and Algebra, Relationships, Measurement and Geometry
**Overall Expectations:** NAV.01, NAV.03, NAV.04, REV.01, REV.02, REV.03, MGV.01, MGV.02
**Specific Expectations:** NA1.01, NA 1.04, NA3.06, NA4.01, NA4.02, NA4.03, RE 1.04, RE1.05, RE 1.06, RE1.07, RE2.02, RE2.03, RE3.04, MG1.01, MG1.04, MG2.01, MG2.02, MG2.03, MG2.04

### Planning Notes

Students may need to review ratios (while creating scale drawings) and formulas for area and volume. The teacher may wish to introduce this activity by brainstorming a list of criteria for a building's base shape (foot print) that would be important to a designer. These could include: size of building lot, height restrictions, by-laws re: distance of the building from the lot line, any natural obstructions on the lot, etc. Before students work on the problem, they will need a discussion of the appropriate scale on their graphs for the large numbers that will come out of the activity.

### Prior Learning

*Patterning and Algebra Grade 7 and 8:* Recognize patterns and use them to make predictions; use the concept of variable to write equations.

*Data Management and Probability Grade 7 and 8:* Identify and describe trends in graphs, using informal language to identify growth, clustering, and simple attributes; know that a pattern on a graph may indicate a trend.

*Measurement Grade 7 and 8:* Apply the volume formula to problem-solving situations involving rectangular prisms.

### Teaching/Learning Strategies

**Student Activity:** Students will work in heterogeneous groups of three but will submit individual reports to the following problem.

### A Design Problem

A designer is hired to create a multi-storied building for a client who wants to open a business for people to play laser tag. The lot is 100 m × 100 m and has a large old tree exactly in the middle. This tree is to be preserved with an open space of at least 5 m in all directions from the centre of it. Town bylaws require a fire route of at least 10 m width along the two sides and back of the property. Each storey is to be 4 m high, have a flat roof, and have the same footprint as the base. The client wants the footprint of the building to have an area of 4750 m.

1. Create scale drawings of 3 building footprints which satisfy all of the constraints.

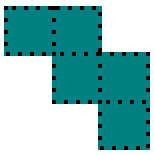
2. For each of the three footprints, how does the volume of the building change as the number of storeys increases? Explain the relationship between the height and volume of each building using a table of values, finite differences, a graph, in words, and using an equation with variables H and V.

**Note:** Teachers may suggest alternative purposes for the building and adjust the design constraints accordingly. For example, if it were to be an office building, one design objective could be to maximize the number of corner offices. If it were to be a movieplex, the design task would be to provide the maximum number of cinemas with good viewing proportions for a specified number of people with adequate adjoining lobby space and access.

3. Should you connect the points on your graphs? Explain your reasoning.

4. How would the graph of volume versus height change if the area of the footprint was 2000 $m^2$ instead of 4750 $m^2$? What would be the equation of this new relationship? Can you suggest an area of footprint that would create a graph steeper than your original graph?

**Teacher Facilitation:** The teacher will show students configurations that are allowed and not allowed, as shown below. The teacher should ensure that discussion about discrete vs continuous functions happens while students are working on question #3. The teacher will circulate around the room and provide prompts as needed, or ask students from one group to give quick hints to another group. Each student in each group of three should take responsibility for creating the written submission of one of the footprints for their group. However, encourage the three students to keep using **collaborative skills** and comparing their results.



Configurations allowed:

Configurations not allowed:

**Extension 1:** Students will discuss the following extension to the building design problem in a large or small group, then work independently to prepare a solution.

Suppose that the client can afford the climate control systems necessary for the volume of a 4-storey building with the footprints that you designed. However, the city passes a new bylaw that restricts the maximum height to 3 storeys. Design a building that has the same volume, obeys the new bylaw, and fits on the property with all of the original restrictions, if it is possible. If it is not possible to preserve all of the original restrictions, explain why and suggest a way around the problem.

**Teacher Facilitation:** The teacher may have to lead students to the idea that, as the height of the building decreases, the footprint area must increase to preserve the volume. Students will find that they cannot redesign the building to fit all of the constraints. They may have creative solutions like cutting down the tree, petitioning the town for an exemption from the fire law restriction if they add enough sprinkler systems, etc. Encourage **analytic skills**, supported by appropriate **numeric** and **communication skills.**

## Go to the Max

**Extension 2SStudent Instructions:** The owner has received suggestions from previous clients that a more interesting game would have a floor plan in which the walls have a maximum surface area and would be only 3 storeys high. To satisfy these customers, what would the shape and dimensions of your new footprint be if its area must be 3600 $m^2$ and the walls must be at least 15 m long?

You still have the constraint of the 10 m fire route on three sides, but no longer need to worry about the tree (it has been removed). There should be no blocked paths so that customers can travel freely throughout the building.

Justify your recommendation to the owner with diagrams and calculations of all the models you have tried.

> **Teacher Facilitation:** Teachers may provide each group of students with blocks to build their floor plan models. Instruct students that each block will represent 15 m × 15 m. When blocks are put together their sides must completely overlap. They may not overlap partially or simply at vertices (as shown).
>
> Students will need to use **analysis skills** to realize that in order to maximize surface area they will need to maximize the perimeter. They may discover that increasing the number of corners will improve the perimeter.

## Assessment/Evaluation Techniques

Use the accompanying rubric to assess student work. The criteria in the rubric elaborate on the criteria in the Achievement Chart.

## Accommodations

Provide a summary of the key points from the ratio review. Provide instructions and requirements in different formats.

## Rubric for Activity 6

| Categories | 50-59% (Level 1) | 60-69% (Level 2) | 70-79% (Level 3) | 80-100% (Level 4) |
|---|---|---|---|---|
| Construction of Graphical Model (RE1.04)<br>• organize and analyze data | • sets up axes but labels are missing or scale is incorrect<br>• table or graph is inaccurate or doesn't match | • sets up axes, uses labels but scale is inappropriate<br>• sets up table or graph but may contain minor errors and may forget about discreteness of data | • sets up axes, uses labels and scale which eventually approaches an ideal<br>• sets up appropriate tables and graphs, but may forget about discreteness of data | • sets up axes, uses labels and readily chooses an ideal scale<br>• sets up well-organized tables, graphs data appropriately and recognizes discreteness of data |
| Construction of Algebraic Model (NA4.01,NA3.06)<br>• use algebraic modelling<br>• rearrange formulas | • needs assistance to develop equation of a relationship<br>• depends on group to explain how to get new base area | • develops equation of relationship given graph or statement of relationship using words or symbols<br>• calculates new base area using trial and error | • develops equation of relationship using words and symbols<br>• calculates new base area by inspection | • accurately and consistently develops equation of relationship and explains the steepness factor in this context<br>• uses informal algebra to solve formula for new base area |
| Construction of Geometric Model (MG1.01, MG2.01, MG2.02) | • understands relationship between volume, base area and height once it has been shown<br>• is able to calculate area, volume and perimeter with assistance; weak understanding of concepts | • recognizes some relationship between volume, base area and height but not fully developed<br>• relies solely on simple methods such as counting to calculate area, volume and perimeter | • recognizes relationship between base area, volume and height after investigation<br>• calculates volume, area and perimeter | • quickly recognizes relationships between base area, volume and height<br>• calculates base area, volume and perimeter using efficient methods |

| | | | | |
|---|---|---|---|---|
| Solve and Pose Problems (RE1.07) | • needs coaching for each step | • solves the problem through periodic reassurance from group leader | • can solve the problem after group discussion | • quickly decides on procedures and solves the problem |
| Compare Algebraic Model with Other Strategies Used for Solving the Same Problem (NA.4.02) | • cannot see the connection between the two models | • can make connections between algebraic and graphical model with assistance | • sees the connections between algebraic and graphical model but uses trial and error to check extensions | • makes connections between algebraic model and graphical model including how changes in one affect the other |
| Describe Effect on Graph (RE3.04) | • is unable to identify components of equations and factors that affect steepness | • states components of equations and factors that affect steepness but cannot explain them | • explain components of equations and factors that affect steepness but lacks details and/or has trouble generalizing | • can easily explain all components of equations and factors that affect steepness in context with detail; can generalize |
| Communicate Findings from an Experiment (RE1.06) | • has difficulty following or incomplete solution | • lacks description of solution but most mathematical forms are present | • combines some description of solution with mathematical forms (tables and graphs); not all connections are evident | • combines description of solution with appropriate mathematical forms; logical flow is evident |

## Activity 7: Fold It; Exploring Non Linear Growth
**Time: 75 minutes**

### Description

The students will investigate nonlinear growth during a simple paper folding exercise. The meaning of exponential notation will be reviewed. The zero exponent and negative exponents will be introduced in a context and then practised.

**Strands:** Number Sense and Algebra, Relationships
**Overall Expectations:** NAV.01, NAV.02, REV.01, REV.02
**Specific Expectations:** NA1.01, NA1.04, NA2.01, NA2.02, NA2.03, RE1.01, RE1.03, RE1.06, RE2.05, RE2.06

### Planning Notes

Make paper available to the students. Review meaning of exponents and calculations with exponents, taking the opportunity to have the students practise their skills from grade eight. This could be assigned as homework or a brief introduction to the activity.

### Prior Learning

*Number Sense and Numeration Grade 7 and 8:* Express repeated multiplications as exponents.

*Patterning and Algebra Grade 7 and 8:* Recognize patterns and use them to make predictions; use the concept of variable to write equations.

*Data Management and Probability Grade 7 and 8:* Identify and describe trends in graphs, using informal language to identify growth, clustering, and simple attributes; know that a pattern on a graph may indicate a trend.

### Teaching/Learning Strategies

**Student Activity:** Working in pairs, students will review basic skills with exponents and gather data that relates exponentially.

> **Teacher Facilitation:** Describe this situation to the class:
>
> *When a pastry chef makes croissants, the butter is worked into the dough in layers by repeatedly rolling out and folding it. The many layers of butter melt and separate the layers within the dough during baking to make a croissant that is extremely light and tender. This will be modeled with the following paper folding activity.*

**Student Activity:** Model the layering of a croissant by completing this activity.

1. Set up a data table that has three columns and twelve rows. Column headings are: Number of Folds; Number of Layers; leave the last column heading blank for now. In the first column, under the heading, list the numbers from zero to ten.
2. Gather data by folding the paper and counting the layers. Record this data in the second column of the table.

3. Conjecture a pattern that would describe the data. Rewrite the entries in the second column to illustrate the pattern and record in the third column of the chart. ( a pattern of 2^0, 2^1, 2^2 layers should emerge)

4. Describe the entry in the third column that corresponds to zero folds. Conjecture a hypothesis for powers with this exponent and use a calculator to test this hypothesis using other bases. Report the findings of this investigation and confirm or refute the hypothesis. State a conclusion for this investigation.

5. Create the first differences of the third column and hypothesize what will be true about the graph of the relationship.

6. Plot this relationship on a grid and sketch the curve of best fit.

7. Determine an equation for the relationship.

8. Make tables and graphs for powers of 3 and 10. How do these two graphs compare to the graph of the paper folding model?

9. Now let's go backwards:

   Starting at your largest number of layers, unfold your paper. Pay attention to the relationship between the number of folds and the number of layers. When you get to zero folds, you will be back to one layer. Now imagine you could go one step further back, that is, undo a fold to create your original piece of paper. How many folds would this represent and how many layers?

   Create a table showing a number of folds and corresponding number of layers. Enter the results of this experiment. Express the number of layers as powers of two. Extend your table for several more stages of this pattern.

10. Describe the pattern that you observed for the negative exponents. Test your conjecture by using a similar table for powers of 3 and 10.

11. Complete this statement after examining the patterns in the table:
    *A negative exponent means …*

**Teacher Facilitation:** Students may need help with the idea that going backwards means that there is half a layer. If the students start at five folds, and go down down to 4, 3, 2, and so on, be sure they realize that the number of layers is being halved when you get to fold zero and layer 1, the moving backwards means going to fold negative 1 and layer 1/2; fold negative 2 is layer 1/4.; and so on.

Conduct a full-class discussion to ensure that students made and understood correct meaning of negative exponents. The following class time and homework time should be spent practising exponent **numeric skills** learned today, as well as reviewing skills involving negative bases and substitution into expressions involving exponents.

**Extension/homework activities:**

1. A politician enters a room and shakes hands with one person. The politician and the person then each find someone else with whom to shake hands. Each of these 4 people now finds someone else with whom to shake hands. The handshaking pattern continues until more than 500 people have had their hands shaken. Make a table and graph to illustrate the pattern. Determine an

equation that describes the relationship.

2. Have students answer this famous nursery rhyme question: "As I was walking to St. Ives, I met a man with 7 wives. Each wife had 7 sacks, each sack had 7 cats. Each cat had 7 kittens. Cats and kittens, sacks and wives, how many were going to St. Ives?"

3. Provide paper and pencil practice of exponent skills using whole number bases and exponents that are zero or negative.

### Assessment/Evaluation Techniques

Students' teamwork and independent work can be assessed through observation using the learning skills rubric from Activity 3. Follow-up work on powers could be assessed with a quiz.

### Accommodations

Provide the chart for recording exponential growth.

## Activity 8: Walk This Way

**Time: 75 minutes**

### Description

Activities 3 to 7 have all involved the gathering of data, plotting data and analyzing the relationship that the graph shows. Activity 8 reverses the process. In "Walk This Way", the student is shown a graph and is required to walk in a way that creates data that matches the given graph.

Students will use the Calculator Based Lab (CBL) or Calculator Based Ranger (CBR) to gather distance-time data and continue to study the characteristics of linear relations. The students will gain skill in interpreting distance vs time graphs and predicting the shape of a graph for a given scenario. All discussions should refer to the steepness of the lines (the term "slope" need not be used until the next unit). Students will develop an understanding of the meaning of independent and dependent variables and will represent the points on a line as ordered pairs. These points will represent a discrete sample of data from a continuous function, so the points should be joined. The idea that different pieces of the graph show different parts of the walk should also be discussed.

**Strands:** Relationships
**Overall Expectations:** REV.01, REV.02
**Specific Expectations:** RE1.01, RE1.02, RE1.03, RE1.04, RE1.05, RE2.01, RE2.02

### Planning Notes

- The teacher will need to set up a TI-83 Overhead Calculator and a CBR (or a CBL with Motion Detector) before the class starts. If using a CBL, the DT walk program using R1 will need to be loaded onto the overhead calculator using a Graph Link. The teacher should practise with this equipment before using it the first time with a class.

**Note:** At this point students should not be asked to match an equation to the graph as suggested by the

"Match It , Graph It" program. They can do this activity in the next unit after they have developed the skills to find an equation from 2 points.

- The students could work in groups doing their own investigations if there is enough equipment available.
- The teacher may want to extend the activity by using a remote car for a demonstration or for students to experiment with.
- Prepare the distance - time graphs to be used in class discussion (see extension)
- Prepare a work sheet containing several D-T graphs and questions about the motion for students to answer. Include questions about the graph if some of the conditions were changed, e.g., in graph (i) If Lionel did not stop at A, what would his travel time be if he continued to travel at the same speed? If Lionel wanted to reach his friend's house 2½ minutes earlier, what would his speed need to be after his stop?
- Encourage students to pose their own questions and suggest solutions to them.

## Prior Learning

- *Data Management and Probability Grade 7 and 8:* Identify and describe trends in graphs, using informal language to identify growth, clustering and simple attributes; Know that a pattern on a graph may indicate a trend.

## Teaching/Learning Strategies

**Student Activity:** Students will observe the effect of walking towards or away from a motion detector as they follow the instructions in the program. They will try to match the graph displayed on the calculator by walking in front of the detector.

A volunteer is needed to walk in front of the detector to match a given graph. The other members in the class should "coach" the volunteer. Students can take turns trying to match the graphs. Eventually you will get good matches for the graphs (student coaches will tell the volunteer where to start, how fast/slow to walk, when to stop, walk towards or away from the detector, they should recognize steepness as speed). In this kinesthetic activity, students have the opportunity to retry a walk with the same graph and/or try a new graph. Each walk takes about 10 seconds, so there is time for many trials until the students become adept at matching the graph.

**Extension**: Ask students to predict how the graph would change if the walker were speeding up or slowing down. Verify their responses by duplicating the motion in front of the detector. Students can come up with their own situations and observe the effects on the graph.

i) Lionel is walking to his friend's house.

ii) A train approaching a station is braking to a stop.

*Suggested Homework:*

(i) Students should summarize in their notebooks what they have learned about different motions and their graphs. They should include a separate drawing for each of the following motions: person is stopped; moving faster/slower; towards/away from a point; speeding up/slowing down.

(ii) Sketch the graph of a journey that includes a quick walk, slow walk, some time standing still, moving towards and away from a point. (Describe the parts of the graph using words like steep, flat, time and speed.)

(iii) Answer questions on the work sheet for the remaining graphs.

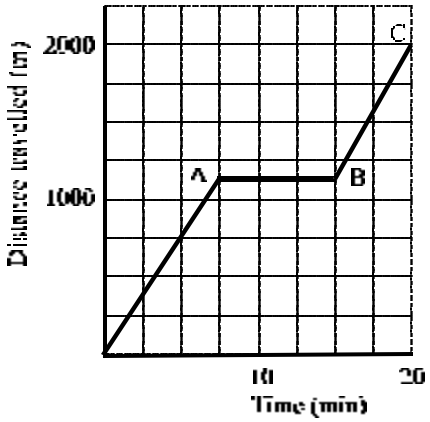
**Teacher Facilitation:** The teacher will inform the students that the detector will take frequent, regular measures of their distances from the detector, creating a discrete sample of their continuous motion.

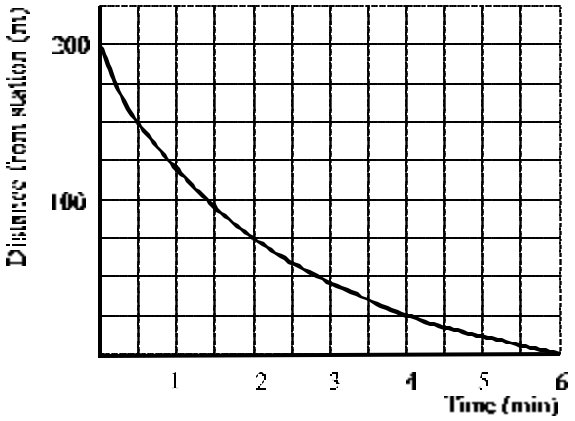
For the first graph (i), the teacher will encourage **analysis skills** by asking questions such as: How long did Lionel take for the complete journey? How long was he stopped? How long would Lionel have taken for the trip if he hadn't stopped? Discuss independent and dependent variables; how the distance changed with the change in time; how the change in steepness related to the walk. Identify some points on the graph as ordered pairs e.g. (t,d) = (5,750) means that a distance of 750 m had been traveled in 5 min. Ask the students Lionel's speeds over different parts of the journey. What further questions can they pose for the problem?

For graph (ii) the teacher will ask questions such as: How far from the station was the train when it started braking? at different times ? How long does it takes the train to stop? What questions can students pose for this scenario?

## Assessment/Evaluation Techniques

The teacher could observe and assess "Learning Skills " and student performance. (see Appendix 1: Using Rubrics to Assess Learning Skills, Activity 3)




A short quiz could be given assessing the students' ability to analyze data from a given graph. Students will be asked several questions for a given graph. They should also be given the opportunity to come up with

questions of their own and answer them. Test each student matching a graph using a CBR and ask the student to explain what they did and what they could do to do a better job. The teacher could assess this individual performance task while other students prepare for the next activity or while they write a reflection journal on this activity.

## References

"Match It, Graph It" from "Real World Math with the CBL", Texas Instruments

## Activity 9: Tell Me a Story
**Time: 75 minutes**

### Description

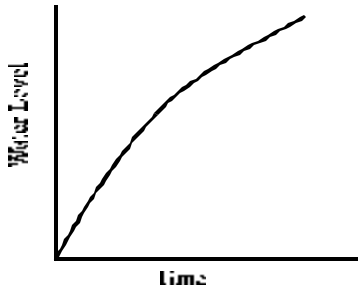
In this activity, students will match different scenarios and their graphs . They will also sketch the graphs for a series of different situations, as well as creating stories that describe given graphs.

### Strands and Expectations

**Strand:** Relations
**Overall Expectations:** REV.03B
**Specific Expectations:** RE3.02, RE3.04, NA1.01, NA 1.02, NA1.03, NA1.04, NA1.05, NA1.06

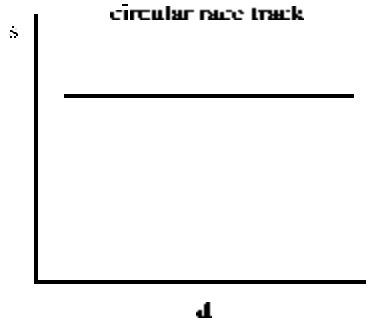
### Planning Notes

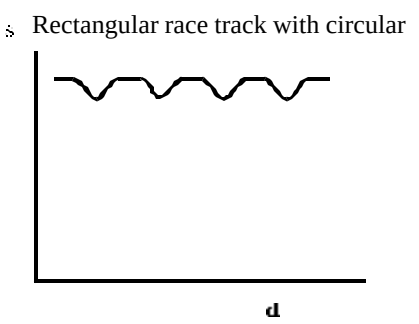
- Gather a variety of activities that have both graphs and stories describing similar situations in which the students are required to match the appropriate story to the graph. For example, a person is travelling at a constant speed, stops for a period of time, and then continues at a constant but slower speed. The graph to match this situation will be a line with some steepness, followed by a horizontal line and then a line less steep than the original line. There are many sources where teachers can find these graphs (see Resources).
- Include graphs which describe a story, containing a flaw that the student will have to correct, and vice versa. For example: The given graph for the situation described above contains line sections at the beginning and at the end with the same steepness. Is there an error in this graph? If so, identify and correct it.
- Include graphs that are continuous, discrete, piece-wise linear, non-linear. (Students have now seen each of these types of graphs in previous activities.)
- Provide students with graphs that model different events/situations (e.g. races with a different number of competitors, car/bus road trips and extend to other situations such as filling a tub or swimming pool, cost of phone calls, population growth, riding a roller-coaster or Ferris wheel, etc.




- For examples, a container shaped like a truncated cone, wider at the top, is being filled with water. What would the graph of the water level over a period of time look like?
- Study these graphs before giving them to students to make sure that you have thought the mathematics that will arise from the stories in the given contexts.
- Encourage detail and creative scenarios that are suggested by the graphs.
- Plan some of the questions that you will ask the class to move them towards higher thinking skills. (e.g., What are some of the things that have been ignored in creating the mathematical models? What changes in the situation would create different graphs ? What would they look like?) For example, what would the above graph look like if the container was wider at the top? Not as wide at the top? If the water was not being poured at a constant rate?

- An extension could be to ask students to sketch speed-distance graphs. For example, sketch the graphs for a car which races around a circular race track and a rectangular race track with circular ends. (See Resources, R2)




Notice how the speed is constant in the circular race track and how the speed varies as the car slows down when going around turns in the other race track.

## Prior Learning

*Number Sense and Numeration Grade 6, 7 and 8:* Add and subtract integers; demonstrate an understanding of and apply unit rates; demonstrate an understanding of ratio.
*Data Management and Probability Grade 7 and 8:* Identify and describe trends in graphs, using informal language to identify growth, clustering and simple attributes; know that a pattern on a graph may indicate a trend.

## Teaching /Learning Strategies

**Student Activity:** Working in groups, students will match up a situation with a graph and explain their reasoning.

For stories that have incorrect graphs, they will have to identify the errors, explain what the error is, and make needed corrections.

Individually they will write stories for various graphs and share some with their group members and with the class. They will also sketch graphs for given situations. For a summative activity (this can be assigned for homework) they could be given the "Bicycle Trip" graph that they can write a story for (the rubrics that will be used to assess their work are included).

**Teacher Facilitation**: The teacher should start the discussion with some of the homework graphs, and review independent and dependent variables. Students should discuss how the distance changed with time. The teacher could have the students calculate the speeds of some of their graph models. **Analysis, numeric,** and graphical **communication skills** are all needed for this activity.

While students are doing the match-up graphs the teacher should encourage students to talk about how the variables change with time and pay attention to the steepness of curves or lines. Start with distance - time so that they can draw upon the previous day's experience.

The teacher should pose some questions that will address higher level thinking.

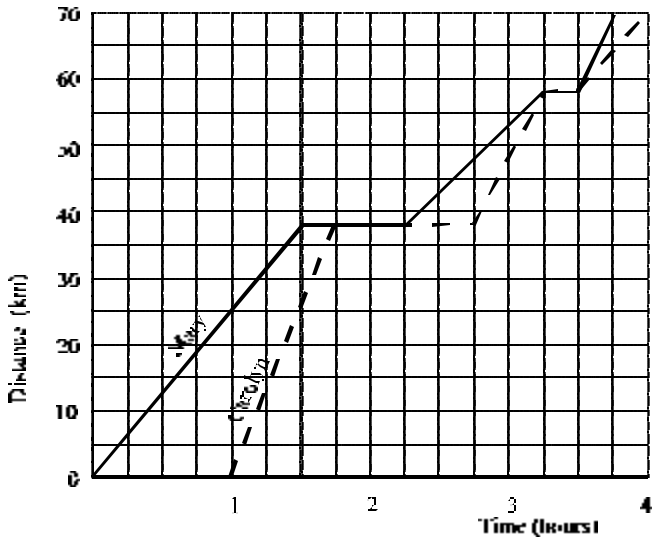
While the students are working in their groups the teacher could assess their ***Learning Skills*** (e.g. Teamwork and Initiative) and performances.

## Assessment/Evaluation Techniques

Use the following graph and develop rubrics and checklists to assess their ability to communicate and apply their knowledge.

## The Bicycle Trip

Mary and Carolyn set out for a bicycle trip. The distance-time graph shows their progress as they reach their destination.




Write a story that describes their trip. This could be a play-by-play sportscast. Details you should include:

- times they were together/apart; stopped; going faster/slower
- possible events explaining the different sections of the graphs
- references to time and distance as well as your calculations of speeds in a narrative style
- comparisons and contrasts

Try to be creative! (in the context of the given graph)

**Accommodations**

Students with writing difficulties could tell the story to a classmate who would write it down for them. Simpler graphs, with fewer sections, can be provided.

### Resources

R1 - Math Mania, Sept. 1991

R2 - Math Mania, Apr. 1992, "Identifying Qualitative Graph" Math Teacher Sept 1994

Balanced Assessment for the Mathematics Curriculum

MARS Project - http://www.educ.msu.edu/mars

**Rubrics for "Tell a Story"**

| **Categories** | **(Level 1)** | **(Level 2)** | **(Level 3)** | **(Level 4)** |
|---|---|---|---|---|
| Knowledge (RE1.05, RE3.04) | needs considerable assistance to calculate speed, distance and time or makes many errors | some calculations for spped, distance and time for both girls are correct | most calculations for speed, distance and time for both girls are correct | all calculations of speed, distance and time for both girls are correct |
| Communication (RE3.02) | has difficulty describing the events illustrated in the graph | some events illustrated in the graph are correctly described | correctly describes most of the events in the graph clearly | describes correctly all the events illustrated in the graph with a high degree clarity and insight; provides additional observations |

## Activity 10 - What Does It Mean? Steepness and Rates of Change
**Time: 75 minutes**

### Description

This activity helps the students to determine the meaning of steepness of a line as a rate of change between two variables that represent a real situation (other than distance/time). The graphing calculator, CBL and Force Probe will be used by the students to determine the relationship between the mass of pennies or other coins, and the number of coins. The students will analyze the data collected in the experiment and attempt to interpret the meaning of the slope as it relates to the independent and dependent variables and describe the rate of change in terms of the mass of 4 coins and the mass of 1 coin. Students will then use their model to answer several questions.

**Strand:** Relationships
**Overall Expectations:** NAV.01, NAV.04, REV.01, REV.02, REV.03
**Specific Expectations:** NA1.03, NA1.04, NA4.01, NA4.03, RE1.01, RE1.03, RE1.04, RE1.05, RE1.06, RE1.07, RE2.01, RE2.02, RE2.03, RE3.01

### Planning Notes

- This activity should be done in groups of 4 so that students can share the equipment.
- The teacher needs to have loaded the program "Pennies", from the Real World Math with the CBL resource book, into the graphing calculators.
- (Graph Link kit required). Each group will need at least one graphing calculator, CBL (Calculator Based Lab) with link cable, a force probe, a Styrofoam cup or an empty yogurt container, string, and 28 pennies (or nickels or dimes).
- An alternative, if there are not enough CBL's and force probes are available, is to do the experiment as a demonstration using an overhead display panel and transfer the data to the students' calculators by using the "LINK" function so that they each have the collected data to work with. Another alternative is to use the “Stretching Pennies to the Limit” experiment with the CBR. Until such time as the CBR or CBL equipment is available, the experiment could be performed using scales borrowed from the science department; and the data would be collected and the graphs prepared by hand.
- Teachers should familiarize themselves with the program and the equipment so that technical difficulties can be anticipated.
- Optional: the calculator has a built-in feature called a linear regression that allows it to compute the line of best fit (use “STAT” key and select “LinReg”). This line can be placed as a function in your “Y=” list using “VARS” and a function register and graphed with the scatterplot using the “GRAPH” key. This is the first opportunity to discuss with the class how we communicate with the calculator using the general $x$ and $y$.

### Prior Learning

*Number Sense and Numeration Grades 6, 7 and 8:* Demonstrate an understanding of and apply unit rates; demonstrate an understanding of ratio.

*Patterning and Algebra Grades 7 and 8:* Recognize patterns and use them to make predictions; use the concept of a variable to write equations.

*Data Management and Probability Grades 7 and 8:* Identify and describe trends in graphs; know that a

pattern on a graph may indicate a trend.

## Teaching/Learning Strategies

**Student Activity:** Students will poke small holes on opposite sides of their Styrofoam cup near the top rim; thread a piece of string through the holes and then tie the ends of the string together to create a small bucket. Suspend the bucket from the force probe; separate their pennies into seven piles of 4 coins each. Run the pennies program on the graphing calculator and follow all instructions.

> **Teacher Facilitation:** Circulate and troubleshoot, helping students with their use of **technology skills** as needed. The most common problem will be that the CBL and graphing calculator will not be linked tightly: push in the link cable firmly and continue with the program. If the results are not satisfactory: students didn't put in exactly 4 pennies, or errors occurred in interpreting instructions; students can run the program again since the data can be collected very quickly.

**Student Activity:** Students should copy the lists (use "LIST" key) from their calculators to their notes and draw their own scatterplot. Students determine a line of best fit and use it to look at first differences, and explain why the mass of 4 coins may not be exactly the same for each measure. They should describe how the variables N, number of coins, and M, mass of the coins, are related and write an expression for the relationship between them. Students should explain how to determine the mass of one penny, and generalize (the steepness of a line is a rate of change) They can use this information to answer questions such as: "What would be the mass of a collection of pennies totaling $3,843.50?" And, "If a container of pennies has a mass of 50 kilograms how many pennies are in the container if the container has a mass of 15 kilograms?" Students can repeat the experiment for nickels or dimes or compare results with groups who used different coins.

**Optional:** The teacher can assist the students in the use of the linear regression capabilities of the calculator in preparation for unit 2. This would also serve as a check for their own lines of best fit. One of the reasons we would want to generalize in this example is to communicate using the calculator.

**Homework:** Students could be given secondary data that shows various relationships between various things such as: the height of some tall buildings and the number of stories in those buildings, (this is a linear relationship but does not start at (0, 0) because first two stories are usually not the same height as the rest of the building); taxable income and tax paid, years and mass of garbage waste, years and population (These last two examples are non-linear, but students could check the rates of change over various intervals and describe the meaning of the steepness of the curve.)

**Extension:**

- Have students perform a spring experiment in which they measure the length of the spring as they hang different masses. This will provide real world meaning to the slope of their lines and describe the relationship in terms of stretch length and mass.
- Provide secondary data for students to work with such as: the number of calories in fast food compared to the number of grams of fat and students will provide real world meaning to the slope of the line and describe the relationship between the variables.
- Analyze data about the strength of a magnetic field related to the distance away from the source.
- Devise an experiment that would predict the time that it would take to count every number out loud to a million or a billion, and describe the relationships that exist depending on the number od digits in the number.

> **Teacher Facilitation:** Circulate and help students as needed. Direct groups to share

information and prompt for generalizations. Provide ideas, equipment, data and direction for extensions to groups who are ready. This extension can draw on **all of the skills** of the unit.

### Assessment/Evaluation Techniques

Teachers could assess Learning Skills by observation (refer to Learning Skills rubrics in the Unit 1 Appendix) and student performance and problem solving and inquiry. They can also assess the presentation or the results of the experiment (written, oral) as well as journal entries of the students' work using a rubric modeled after the Rubric for Assessing Student Presentations, found in Activity 3.

*Quiz:* Provide data in context, from secondary sources, that shows a linear relationship between two variables such as: the length and width of 4-door cars, the mean distance from the sun and the time in years of one revolution around the sun of the planets in our solar system. Have students draw a line of best fit and explain the rate of change of the dependent variable with respect to the independent variable and explain the real world meaning. A non-linear relationship question might also be included in the quiz.

### Accommodations

Provide written steps for calculator use rather than only verbal steps. Limit the complexity of some experiments.

### Resources

"Real-World Math with the CBL System", 25 Activities Using the CBL and TI-82, Texas Instruments.

"Advanced Algebra Through Data Exploration" A graphing Calculator Approach, Key Curriculum Press.

"Proquest."

## Activity 11: Summative Assessment Activity
**Time: 150 minutes**

### Description

This summative assessment package includes two parts: summative performance activities and a summative pencil and paper test for this unit.

### Planning Notes

- This assessment package requires several days to complete. Two performance-based assessment activities (Activities 1 and 2) and one pencil & paper test (Activity 3) are included, to allow for a balanced view of the student's overall achievement on this unit.
- In creating the paper & pencil test, teachers must include questions that provide opportunities for students to demonstrate level 4 performance. Sample questions are available in Activity 3.
- Teachers should have available a variety of the materials and technologies used throughout the unit, so that students will feel free to use familiar items as they complete the activities (graphing calculators, graph paper, string, etc.)

### Prior Learning

The students will have completed the first unit of the grade 9 Academic course.

## Activity 11.1: Summative Performance Assessment

## Time: 75 minutes

**11.1a Teacher Introduction: (10 minutes)** Introduce the activity by discussing uses and functions of a roof. Lead students to brainstorm that the roof of a building is designed to protect the building from the "elements", including heat, cold, rain, snow, drought, and extreme wind conditions. The strength of a roof is very important to the safety of the people who work inside or near the building.

Have a brief discussion on the role of an engineer in the design of roofs, and how an engineer simulates situations that affect the stability of a roof in order to design safe roofs.

Use the blackline master that has been provided (titled “Testing Paper Roofs”) to present the situation that the students will model. Share with students the rubric that will be used to assess their performance.

**11.1b Student Activity: (30 minutes)** The students begin the activity in groups of 2 or 3, modelling the strength of a roof. Students collect the data from the experiment. They will prepare to make a variety of conclusions based on their models.

> **Teacher Facilitation:** Before beginning the activity, ensure that students are aware that they will collect data in groups, but will submit individual work. Circulate around the classroom, prompting students as needed. Teachers can be assessing problem solving through observation, noting what students say, how they explore and hypothesize, etc.

**11.1c Student Activity: (30 minutes)** Individually, students organize, display and make inferences from the data. Students make and justify their prediction for collapsing weight of a roof of 100 thicknesses of paper, and present their findings in writing. Each student submits a complete solution to the activity.

> **Teacher Facilitation:** The problem posed in the academic version of this activity is worded in a more

open-ended fashion than in the applied. Teachers should choose the amount of structure appropriate to the needs of their students  Circulate around classroom, prompting students as needed. Assess the students' written submission using the rubric given below, or develop one of your own through collaboration with colleagues:

| | **Level 1** | **Level 2** | **Level 3** | **Level 4** |
|---|---|---|---|---|
| Collect, organize and display data using appropriate techniques | • displays data with many errors or needs teacher support to display data | • displays data with some inaccuracy (scale on graph, intervals) | • displays data accurately on table and graph | • displays data accurately on table and graph with considerable creativity |
| Analyze data and make inferences from data | • unable to analyze data without assistance<br>• major errors in analysis<br>• doesn't recognize differences between linear and nonlinear scatter plots | • makes analysis with some prompting<br>• some errors in analysis<br>• has difficulty recognizing differences between linear & non linear scatter plots | • accurate analysis with only minor errors<br>• consistently recognizes linear & non-linear situation<br>• answers the simulated question. | • complete and accurate analysis<br>• consistently recognizes linear & non linear situation<br>• generalizes connections to roofing & engineering |
| Communicate the findings of an experiment | • communicates with limited clarity<br>• justifies conclusions with limited justification | • communicates results with moderate clarity<br>• justifies conclusions with moderate effectiveness | • communicates clearly<br>• justifies conclusions reached for the simulation with considerable effectiveness | • communicates and justifies clearly and concisely with high degree of effectiveness<br>• poses "What if?" problems related to roofs or other similar contexts |

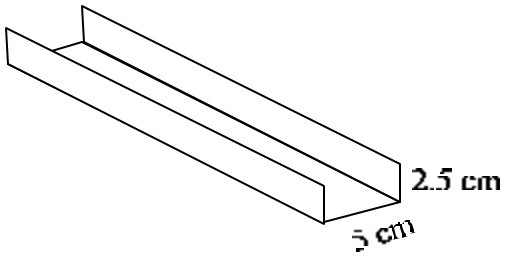
**11.1d Student Activity: (homework)** Students consider other roofing situations that could be modelled or simulated. These could involve changes to the experimental method suggested today (e.g., Would it make a difference to the number of coins needed to collapse the roof if the coins were spread evenly over the roof

as opposed to concentrated in one area? Would the number of layers be different if a different type of paper were used?), or changes to the parameters of the experiment (e.g., What changes would occur if the roof were sloped?) This will form a basis for discussion before tomorrow's activity.

## Activity 11.1 : Flat Roofs - How Strong Are They?

In 1998, a severe ice storm hit Eastern Ontario and Quebec. Many houses were damaged because the roofs could not support the weight of the heavy snow and ice. Determining the strength of a roof is an important consideration in building a house.

Many factories and offices have flat roofs. In this problem, you will do an experiment to simulate the weight-bearing capability of a flat roof.




1. Send the materials handler in your group to pick up the following items:
- a small light-weight container and a collection of identical weights (e.g., washers, nails, coins) to test the load that your "roof" will bear
- six 28 cm by 10 cm strips of paper to simulate roofs of various thicknesses
- two supports of the same height to simulate the walls.

2. Stack the strips of paper neatly, then fold the stack carefully, making a narrow lip on each side, as shown. Each strip simulates a layer of the roof.

3. Suspend a single-layer roof between the two supports. The roof should overlap each support by the same amount on both sides. Place the container in the centre of the roof model.

4. Add weights to the container, one at a time, until the roof collapses. Record your results in a table that shows the number of layers in the roof and the number of weights needed before the roof collapses.

5. Create a roof with one extra layer.

6. Repeat steps #4 and #5, until you collapse a roof of 6 layers.

7. Analyze the data that you have gathered to predict the number of weights needed to collapse a roof with a thickness of 100 layers. Justify your prediction.

## Activity 11.2: Summative Performance Assessment (75 minutes)

**11.2a Teacher Introduction: (10 minutes)** Discuss the other roofing situations that students thought could be modeled using paper and pennies as in yesterday's investigation. Brainstorm such situations as using a different strength of paper to model sturdy or flimsy roofing materials; mixing types of paper to model cement mixed with steel; adjusting the length/width/area of the paper to model roof sizes; sloping the roof; etc. Be sure to mention that the idea of sloping will be developed further in the next unit. Students could further investigate roof structures (via Tech. Dept., Internet, an architect, etc.), in particular why some are flat, some steep. This will be discussed at the beginning of Unit 2.

**11.2b Student Activity: (30 minutes)** In groups of 2 or 3, students will formulate a hypothesis about the relationship between roof length and collapsing point. They will write their hypothesis down before they design and carry out an experiment that models the situation. Students collect the data from the

experiment. They will prepare to predict the theoretical collapsing weight of a paper that is 2 cm long, since this would be too short to actually model.

> **Teacher Facilitation:** Before beginning the activity ensure that students are aware that they will collect data in groups, but submit individual work. Circulate around the classroom, prompting students as needed.

**11.2c Student Activity: (30 minutes)** Individually, students organize, display and make inferences from the data. Students make and justify their prediction for collapsing weight of a 2 cm long piece of paper, and present their findings in writing. Students submit individual solutions to the activity.

**Teacher Facilitation:** Circulate around classroom, prompting students as needed. Assess the students' written submission using the rubric given above, remembering that:

a) yesterday's activity was a linear relation and today's activity is non linear.
b) alter the rubric to include "design a model " with "collect and organize Data …"

## Activity 12: Paper & Pencil Test
## Time: 75 minutes

### Planning Notes

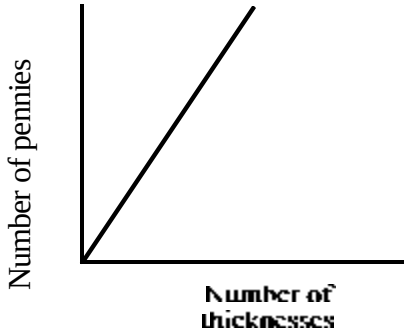
The following samples are questions that could provide students with the opportunities to demonstrate a full range of performances. It would be reasonable to include several of these (or similar questions) in addition to the skill and knowledge questions that are common on a test.

**Student Activity:** Individually, students will write a test that examines numeric and algebraic skills as well as processes studied over the course.

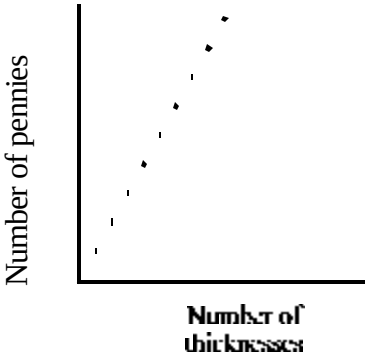
> **Teacher Facilitation:** Circulate around the room as students write the test, prompting students as needed. Advise them that you are recording any prompts that you provide. Write all prompts on the test paper in the space where the student is attempting the question. For ease of recording prompts, use an alternate coloured pen to write the formula, hint, diagram etc. The degree to which the student requires prompts can be reflected in your assessment. Encourage students to be as independent as possible.

## Sample Questions for Pencil and Paper Test

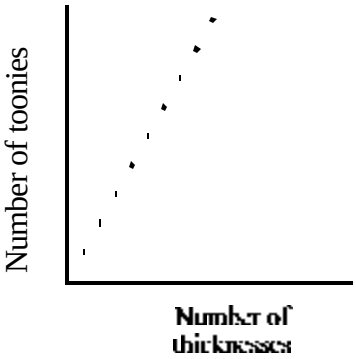
1. Alexa's group modelled the collapsing weight for various thicknesses of roofs, using a paper cup. Alexa's group drew this graph from their investigation.




How would the graph differ if your group had used a heavy cup instead of a paper cup? Sketch the graph that would result from your experiment. Alexa's original graph using a paper cup is drawn as a dotted line as a reference graph. Explain your reasoning.




How would the graph differ if your group had still used a paper cup, but toonies ($2 coins) instead of pennies? Sketch the graph that would result. Alexa's graph using a paper cup and pennies is drawn as a dotted lineto be used as a reference line.Explain your reasoning.

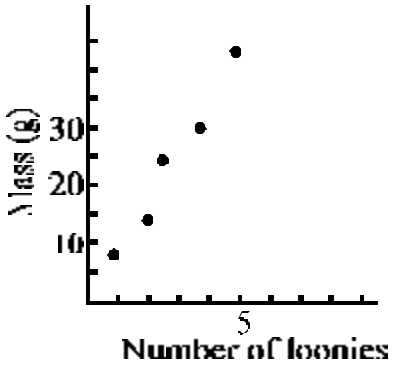



2. Jaime recorded the following data in a scatterplot that graphs the relationship between the number of loonies and the mass of the loonies.

   a) Draw a line or curve of best fit. Explain why you drew a line or a curve.

   b) Determine an equation for the relationship

   c) Predict the mass of 100 loonies



3. Sue drew a graph that displayed the data she collected from a paper folding investigation. She determined that the equation of her graph was $L = 2^F$ where F is the number of folds and L is the number of layers of paper . Which of the following points will lie on the graph? Explain your reasoning. Rewrite the points that are not on the graph with values that will place them on the graph.

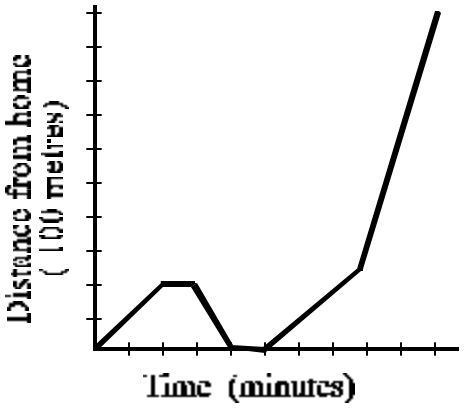
a) (F, L) = (3, 6)

b) (0,0)

c) (4,16)

d) (25, 5)

4. The graph to the right depicts Maggie's trip to school one day. Write a story describing her trip. Be sure to use appropriate times, speeds and distances in your description.




5. Two runners, Ari and Josh ran a marathon race of 30 km, with each runner progressing at a constant rate. Ari ran the course in 150 minutes, and Josh took 180 minutes.

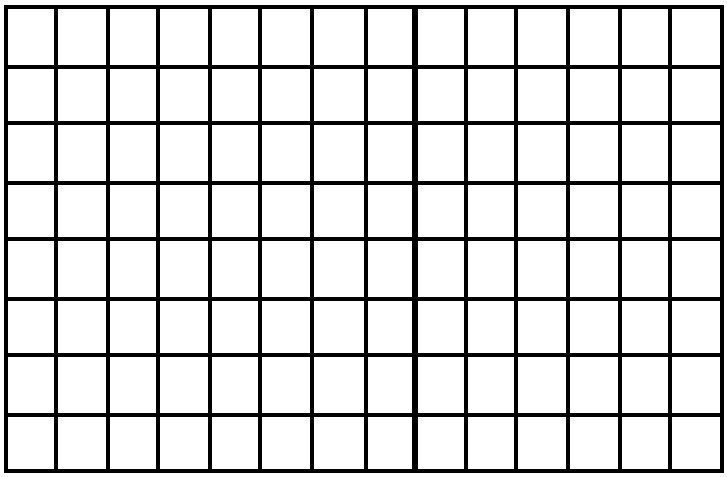
a) Draw two graphs on the same grid, one to represent each run.

b) Write equations that represent the distance, d, that each runner has run after t minutes.

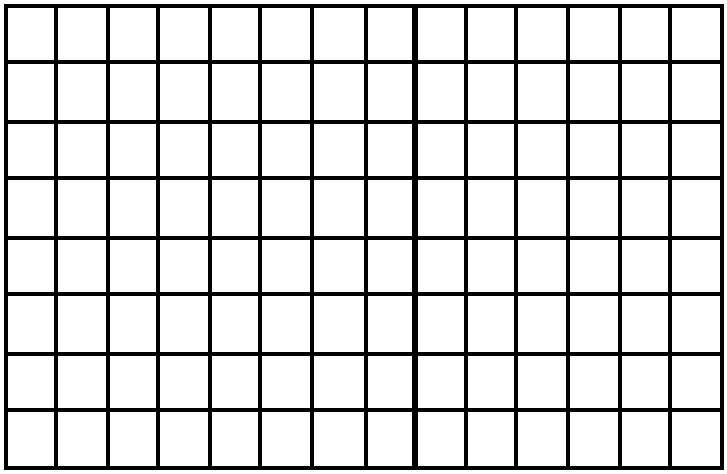
c) When will the faster runner be ahead of the other runner by 1 km.?

d) How far ahead is the faster runner when he finishes the race?

e) If Josh were to speed up at the 60 minute point of the race, how fast would he need to run to finish the race at the same time as Ari?



f ) No one ever runs a marathon race at a constant rate. Think of a more realistic scenario for Ari. Draw a new graph for Ari to represent this scenario. Explain in detail each part of the graph.



6. Form a hypothesis about any relationship not investigated yet during this course. State what data you will gather and how you will gather it, to support or refute your hypothesis. (Note: This question should be shown to students the night before.)

## Appendix 1: Using Rubrics to Assess Learning Skills

In addition to course expectations, teachers will report on students' learning skills. These skills are a separate evaluation component and are no longer to be included in the overall mark. Each behavior on the report card should be observed at least three times per reporting period per student. Tracking tools are needed in order to evaluate students' learning skills efficiently and effectively.

The rubrics shown here offer descriptors of the desired behaviors at each level for two of the learning skills. A rubric for inquiry and problem solving could be done the same way. No student will be exhibiting all of the behaviors at any given time. Rather, the rubrics offer a variety of indicators for each level. After choosing the skill to be observed, the teacher could carry the rubric, across the top of a class list, on a clip board while circulating during a student activity. The date on which the skill was observed would be entered in the column representing the appropriate level next to the student's name. It may not be possible to assess each student on any one day. Teachers should select rows of the rubric to focus on in a recording period. The rubric is meant to be used to collect observations throughout a reporting period. The levels in the rubric could correspond to those on the report card as follows: U • Level 1; S • Level 2; G • Level 3; E • Level 4

As with any rubric, it is important to inform the students about the assessment criteria in advance.

## Independent Work

| 1<br>**With considerable assistance** | 2<br>**With moderate assistance** | 3<br>**With minimal assistance** | 4<br>**Independently** |
|---|---|---|---|
| • Begins task with prompting<br>• Reads and follows instructions with limited effectiveness, requires help<br>• Pursues alternate strategies if initial one does not result in a solution, with considerable help | • Begins task with some prompting<br>• Reads and follows instructions with moderate effectiveness, sometimes requires help<br>• Pursues alternate strategies if initial one does not result in a solution, with help | • Begins task promptly most of the time<br>• Reads and follows instructions without help, with considerable effectiveness<br>• Pursues alternate strategies if initial one does not result in a solution with some prompting | • Always begins task promptly<br>• Reads and follows instructions without help, with a high degree of effectiveness<br>• Actively pursues alternate strategies if initial one does not result in a solution |
| • Does not check solutions, unless prompted, requires considerable help to make corrections | • Rarely checks solutions (if available), makes corrections with help | • Frequently checks solutions (if available), and makes corrections, sometimes with help | • Always checks solutions (if available) and makes corrections without help |
| • Does not refer to class notes, text, other resource materials before seeking help | • Rarely refers to class notes, text, other resource materials before seeking help | • Frequently refers to class notes, text, other resource materials before seeking help | • Always refers to class notes, text, other resource materials before seeking help |
| • Does not pursue extensions | • Pursues extensions, with prompting, requires help | • Seeks extensions with some prompting | • Seeks extensions without prompting |

## Teamwork*

| 1<br>Is passive or unproductive influence | 2<br>Needs encouragement to work productively | 3<br>Works productively without prompting | 4<br>Is a leader |
|---|---|---|---|
| • Waits to be given a role | • Chooses own role with little consideration of talents of other group members | • Helps to assign and clarify role of each group member after choosing own role | • Helps to assign and clarify role of each group member to the optimize the performance of the group |
| • Meets the expectations of own role with limited effectiveness | • Meets the expectations of own role with moderate effectiveness | • Meets the expectations of own role with considerable effectiveness | • Meets the expectations of own role with high degree of effectiveness |
| • Must be reminded to remain on task most of the time | • Helps group remain on task with moderate effectiveness | • Helps group remain on task with considerable effectiveness | • Is on task at all times |
| • Frequently uses exclusive language or negative tone of voice, discourages participation of some group members | • Must be reminded to remain on task some of the time | • Is on task most of the time, does not require external prompt to return to work | • Helps group remain on task with high degree of effectiveness |
| • Assumes a passive role, allows other group members to regulate activities | • Encourages equal participation of all group members some of the time, frequently uses exclusive language or negative tone<br>• Sometimes assumes a passive role | • Encourages equal participation of all group members most of the time, occasionally uses exclusive language or negative tone of voice | • Encourages equal participation of group members at all times through the use of appropriate language and tone of voice |
| • Continually attempts to dominate the group | • Frequently attempts to dominate the group | • Occasionally attempts to dominate the group | • Does not attempt to dominate the group |
| • Rarely offers constructive criticism | • Infrequently offers constructive criticism | • Sometimes offers constructive criticism | • Frequently offers constructive criticism |

| 1<br>**Is passive or unproductive influence** | 2<br>**Needs encouragement to work productively** | 3<br>**Works productively without prompting** | 4<br>**Is a leader** |
|---|---|---|---|
| • Is often the cause of group conflict<br>• Relies on other group members to resolve difficulties | • Helps deflect group from conflict with limited effectiveness | • Helps deflect group from conflict with considerable effectiveness<br>• Asks clarifying questions | • Helps deflect group from conflict with high degree of effectiveness<br>• Asks clarifying or extending questions |

***** Group tasks not only allow teachers to assess "team work" but also to "shine a light" on student problem solving.

# Unit 4: Putting It All Together - Summative Assessment Activities

**Time: 15 hours**

**Unit Description**

This unit is a final assessment in grade 9 mathematics and will give the teacher a vision of how rich problems and performance tasks can be used to assess student performance of the expectations within the course. These assessment activities follow the pattern of the summative assessments in the previous units. Both traditional and performance assessments are essential to comprehensively address the variety of overall expectations in the course. A paper and pencil exam alone is not a sufficient summative assessment.

The form and substance of this summative assessment unit should be shared with the students near the beginning of the course. There are four components to the unit.

The first component is a performance task in the form of a "Mystery." When students do open-ended, multi-dimensional work that requires them to choose and use problem-solving strategies, we are inviting students to show what they know and can do mathematically. The performance task that is suggested here is a solution to the mystery "Who's Got the Gecko?", a scenario where the gecko is stolen from the science workroom. However, this scenario could be adapted to another situation. The focus is on building a series of connected activities that allows for assessment of the overall expectations in context. Each activity is built on course expectations from the previous units, and in context, presents a different piece of evidence that students will use to build a case to determine a profile of the suspect.

The second component of this unit contains an optional fifth performance task and presentation of student work as a culminating event, a celebration of learning. Teachers might use little or most of the 5 hours allotted for this second component, as described in activities 5 and 6, depending on the needs of the class.

The third component of this unit is 1.5 hours allotted for review of skills and knowledge before the final exam.

The fourth component of this unit is a 1.5 hour paper and pencil exam that includes the knowledge and understanding of the overall expectations not addressed through the performance assessment activity of the first component of the unit.

**Strand(s) and Expectations**

**Strands:** This unit covers all strands of this course.
**Overall Expectations:** NAV .01; NAV .02; NAV .03; NAV .04; REV .01; REV .02; REV .03; AVG .01; AVG .02; AVG .03; MGV .02; MGV .03

### Activity Titles

| | | |
|---|---|---|
| Activity 1: | The Suspect's Footprints - Making Inferences from Evidence | 2 hours |
| Activity 2: | The clue is in the water - Using loss of volume, and rates of water flow to determine the time of the crime | 1 hour |
| Activity 3: | Getaway Speed - Using properties of linear relations | 2 hours |
| Activity 4: | Where did he go? - Using geometric properties to provide clues | 2 hours |
| Activity 5: | (This activity is an optional extension for students who have additional time or want to work on their own time) Creating their own Clues - Applying mathematics in an original setting to create a new clue | 2 hours |
| Activity 6: | Presenting the case | flexible, depending on the form of presentations |
| *Time for Exam Review (Use this time when appropriate) | | 1.5 hours |
| Activity 7: | Final Exam | 1.5 hours |

### Prior Knowledge Required

Students will need to have the knowledge gathered from Units 1, 2, and 3 of the course. This unit contains activities with direct links to previous units. Students will also need group learning skills, familiarity with self-reflection and peer assessment, and the expertise of communicating detailed, justified solutions to problems .

### Unit Planning Notes

- "Who's Got the Gecko?": The students will work in groups through the five activities leading to Exhibits A through D(E) to help them create a profile of the suspect. For purposes of timing, one hour of activity should probably take one (70 - 75 minute) class period, allowing for set-up, clean-up time. Students will be using technology that has been used throughout the course which includes spreadsheets, graphing calculators and dynamic geometry software.
- Each activity should be structured to allow time for group work and time for writing submissions individually. The sixth activity will be the presentation and justification of the profiles that each group has created. Students will be assessed individually. This will be achieved through teacher observation and also through the assessment of each of the Exhibits that students will be submitting on a regular basis in their individual Case File Folder, which is a portfolio of their work. It is suggested that the teacher conference with the students prior to their presentation. During the presentation, it might be helpful if each student is responsible for presenting the argument for a different piece of evidence.
- Teachers may want to adapt the gecko scenario or choose another project as the summative activity. Other suggestions include:
    - Investigating costs of different plans of cellular phones and the calling ranges of cell phones
    - The design, rental fees, heating costs related to volumes, roof structuring, capacity, parking lot design etc. of a local stadium
    - Designing a sprinkler system through an investigation of water flow, volumes and surface area of pipes, costing graphs, sprinkler coverage, etc.

- When designing this summative unit, teachers should keep in mind that the activities address a broad range of expectations in the course and should do so in a realistic context.

## Teaching/Learning Strategies

- This unit requires flexibility of timing and structure so that students are engaged in meaningful tasks. Since much of the unit is a performance assessment, the teacher needs to continually circulate, observe and prompt when needed.
- The teacher must allow time for both group work and individual work so that students can prepare their submissions for their case file. All of the work should be done in class so that the work is the student's own, with the exception of Exhibit E.
- Since most of the work will be done in class, there will be little homework related to the "Mystery" project. Homework should consist of review material for Activity 7 which is a test of their knowledge of the expectations not covered by the performance tasks.
- Teachers may also need to spend a period partway through the project so that students can "catch-up" if they have missed a period or part of a period. This could also be a good time to review and take up some of the homework questions that have been assigned.

## Assessment/Evaluation Techniques

The focus of assessment in both the performance task and written exam is on overall expectations.

Assessment will occur throughout the performance task during each activity and at the end of the unit as students present their "suspect profile" to the class. The "suspect profile" could take a variety of forms: wanted poster, a skit to role play the crime, a Crime-Stoppers Ad, a radio, newspaper or television bulletin about the crime. It is expected that students use the evidence gathered from the five activities (Exhibits A - E) to make a convincing argument for their profile of the suspect in their presentations. (An alternative to creating a suspect profile is to give students a list of possible suspects and their characteristics. Students would make a case for choosing one of the suspects as the most probable culprit. Depending on the nature of the class, students may be more comfortable with this alternative.) Discuss these forms of presentation with the students, in advance of the assessment activities, to encourage a variety of presentations.

Assessment of the performance task ("Who's Got the Gecko?") will occur through observation, checklists, rubrics or informal notes; submissions using well defined rubric; conferencing notes; student reflections in their work log; as well as peer and teacher assessment of the final presentations. Two general rubrics, Observing Students at Work and Assessing Student Submissions, found in the Appendix to Unit 4, can be modified and used as assessment tools throughout this Unit.

Although students will work in groups to brainstorm and investigate, their work will be assessed individually**.** The work log in their Case File Folder should have daily entries, using a process like that developed in Unit 3. The teacher may want to pose key questions that they could address in their work log which would contain comments about how the group worked together, what they found most frustrating or rewarding, where they have used this type of investigation before, how the pieces of

evidence are fitting together, what they already can tell about the suspect and what other kinds of data detectives gather in an investigation that require math in their analysis.

Once this unit has been completed, the teacher will need to gather all of the assessment pieces and combine these in a form based on the Achievement Chart.

As teachers complete their first year of using this performance assessment, they should gather exemplars to help determine and describe levels of performance in following years. Teachers should also work together, as a department or as a region, to refine rubrics, to develop other assessment tools and to discuss assessment criteria. A consensus among teachers about the meaning of the criteria and how they are applied builds the foundation for scoring consistency. All assessment tools should be shared with the students before the students engage in the activities.

The paper and pencil test (or exam) of knowledge is a common form of assessment. However, teachers should design this test so that it includes questions that provide an opportunity for students to demonstrate a full range of performances.

## Resources

Shilgalis.W. “Finding Buried Treasures - An Application of the Geometer’s Sketchpad.” The Mathematics Teacher. 91.2 (February, 1998) pp. 162 - 165

Fendel, D. & Resek, D., Interactive Mathematics Program. Toronto 1997 McGraw-Hill Ryerson Limited

## Activity 1 - The Suspect's Footprints - Making Inferences from Evidence

### Time: 120 minutes

### Description

In this activity, students will be given Exhibit A, a photocopy of a set of footprints found outside the science workroom. Students will work in groups with graphing calculators and/or spreadsheets and use data collection to establish a correlation that will help them determine the height (and perhaps other information) of the suspect. Students will be expected to apply their knowledge from the first unit (i.e. determining relationships between variables, by collecting and analyzing data, to make conjectures about the suspect in a crime).

### Strands and Expectations

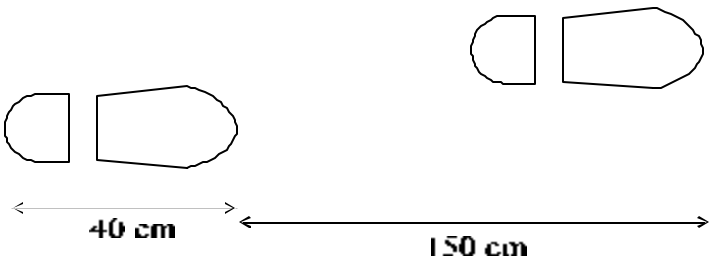
**Strand:** Relationships
**Overall Expectation:** REV.01
**Specific Expectations:** RE 1.01, RE 1.02, RE 1.04, RE 1.05, RE 1.06, RE 1.07, RE 2.02, RE 2.03, RE 3.01

### Planning Notes

The teacher should discuss the entire performance assessment project before beginning. This would include informing students of the types of clues they will receive, the methods of assessment and expectations assessed, the creation of individual case files, and the teacher's expectations for the final presentation. Students could share in the preparation or fine-tuning of the rubrics and checklists.

Graphing calculators, graphing software, and spreadsheets should be available for students. If the TI-83(plus) graphing calculator is used, familiar features such as creating lists, graphing data, and using linear regressions would be used. Graph paper and metre sticks should also be available.

A photocopy of the footprints will need to be created, including an appropriate scale. A sample is included:




### Prior Knowledge Required

Students will need to know how to work constructively in groups, and use the concepts from Unit 1. Students will also need to draw conclusions based on their findings and to justify their reasoning. This has been emphasized throughout the units.

### Teaching/Learning Strategies

**Student Activity:** In groups of 5: Each group receives a copy of the footprint evidence. They will discuss how they will use this data to help them create a profile of the suspect, specifically to determine the approximate height of the suspect. To do this, students will conduct an investigation to gather data to relate heights and foot (or stride) lengths. They can use graphing calculators or graphing software to determine a line of best fit and a relationship between the two variables in a variety of ways (equation of the line, linear regression, extrapolation, interpolation, or determining a correlation factor). This correlation will help them determine the height of the suspect.

> **Teacher Facilitation:** Teachers will need to circulate, keep observation notes, assist with technology and prompt when necessary.

### Assessment/Evaluation Techniques

Teachers will be assessing this activity through observation of the process and the submission of Exhibit A by each student. The two sample rubrics found in the Unit 4 Appendix may be used to assess this activity.

## Activity 2 - The Clue is in the Water - Using Loss of Volume and Rates of Water Flow to Determine the Time of the Crime

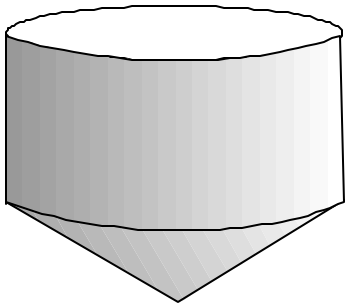
**Time: 1 hour**

### Description

In this activity, students will be given Exhibit B, which includes the dimensions and shape of the gecko's water bottle, the time that it was refilled, the depth of the water drained, and the length of time it takes the gecko to drink all of the water in the bottle.

### Strands and Expectations

**Strand:** Measurement and Geometry
**Overall Expectation:** MGV.02B
**Specific Expectations:** MG2.02, MG2.04



**Planning Notes**
Provide instructions for students that includes a picture or description of a water bottle that is cylindrical with a conical bottom, as shown.

### Prior Knowledge Required

Solving rate problems and concepts from Unit 3.

### Teaching/Learning Strategies

**Student Activity:** Initially, students will be working in their groups but time will be allowed for

individuals to write solutions. Here is the evidence:

The gecko has a water container in its cage that *could be modeled by combining an inverted cone of height 3 cm topped by a cylinder that is 4 cm high with a diameter of 6 cm.* The animal can drink a full dispenser of water in 8 hours. The gecko was given a full container of water at exactly 8:00 am and the police noted that the water level was down 2 cm, determine the approximate time at which the crime occurred.

**Teacher Facilitation:** The teacher should circulate, observe and prompt. Encourage students to draw a diagram of the water container. Students may want to consult resources to find appropriate formulas.

### Assessment/Evaluation Techniques

The teacher will observe students when they are working in groups and assess their skills in teamwork, use of information, initiative, and independence. This can be tracked using a checklist- seating plan sheet, or the rubric, Observing Students at Work, found in the Unit 4 Appendix. Written submissions will allow teachers to assess the students' ability to: communicate with clarity and justification, use correct mathematical symbols, follow the steps of an inquiry/problem solving process, make an argument, and apply concepts and procedures to solve problems in unfamiliar settings. The overall expectation "solving problems involving the volume of three-dimensional objects," MGV.02, can be assessed using a modified version of the rubric, Assessing Student Submissions, found in the Unit 4 Appendix.

## Activity 3 - Getaway Speed - Using Properties of Linear Relations
## Time: 2 hours

### Description

In this activity, students will be given Exhibit C and will be using their skills in Analytic Geometry to determine the speed of the suspect. They will conduct experiments measuring people running at various speeds to determine some additional information about the thief. Using their skills in Relations they will add attributes to their suspect profile.

### Strands and Expectations

**Strands: Analytic Geometry, Number Sense and Algebra, Relationships**
**Overall Expectations: AGV.03; NAV.01; REV.03**
**Specific Expectations: AG2.01; AG3.05; NA1.03; RE3.03**

### Planning Notes

Provide stop watches, measuring tapes, and/or a CBL/graphing calculator with motion detector programmed to collect measures of distance vs. time.

### Prior Knowledge Required

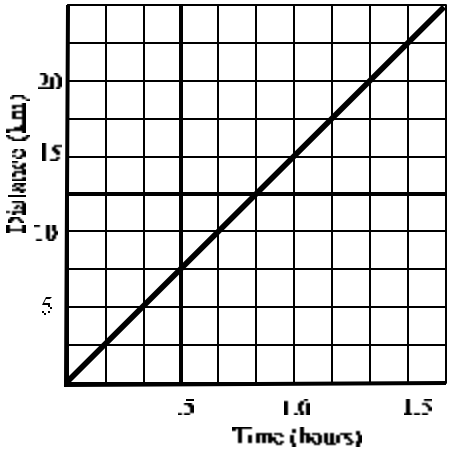
Concepts from Unit 1 and 2, solving rate problems

## Teaching/Learning Strategies

**Student Activity:** Students will work initially in their groups and then individually to write up their submissions. Students will be given the following evidence:

*On the day of the crime a motion detector was left on top of the gecko's cage. It faced the door and captured the movements of someone in the vicinity of the cage. When the police examined the data collected with the detector they were able to determine the motion of someone who might be the thief.*

(i) Using the given graph, calculate the speed of the thief (include reasonable units in your rate).




(ii) If the front door of the school were 150 m from the science room, approximately how long would it have taken the thief to leave the building?

(iii) Using your solution to (i) above, and your choice of tools, determine some information about the thief. Could it have been someone in your class? An older person? An athlete?

(iv) Write a possible scenario of the events that occurred in the room on the day of the theft. "Tell a story" and sketch a possible graph showing the motions of the events you have described relative to the detector.

(v) Submit work to case file.

**Teacher Facilitation:** Students may need some prompts for part (iii) such as, "What is a reasonable running speed for someone in this class? Would using a CBL/CBR help? Could it have been an older person? An athlete?" For part (iv), "What are some possible activities in the room before the theft? Have you shown an appropriate slope to correspond to the time of the theft? Does your graph clearly show when the theft occurred?"

### Assessment/Evaluation Techniques

The teacher will observe students when they are working in groups and assess their skills in teamwork, use of information, initiative, and independence. Teachers may wish to use or modify the rubrics for independent work and teamwork, provided in the Unit 4 Appendix, for this purpose. Written submissions will allow teachers to assess the overall expectation AGV.03, using the rubric for observation provided in the Unit 4 Appendix, along with the following rubric.

| Categories | 50-59% (Level 1) | 60-69% (Level 2) | 70-79% (Level 3) | 80-100% (Level 4) |
|---|---|---|---|---|
| **Application of their concept of slope**<br><br>• solve problems, using the properties of linear relations | • needs help to recognize and apply their concept of slope in making an argument in a new context | • is moderately successful in applying their concept of slope to make weak arguments in a new context | • recognizes and applies their concept of slope to make reasonable arguments in a new context | • recognizes and applies their concept of slope to make valid and convincing arguments in a new context |

## Activity 4 - Where Did He Go? - Using Geometric Properties to Provide Clues

### Time: 2 hours

### Description

In this activity, students will play the role of the kidnapper in preparing a cryptic note to police which identifies where the gecko can be found. Students will use a grid with important landmarks identified and the location of the gecko, and dynamic software to design their note. Once the kidnapper's note had been written, the students will be shown a map which fits with the grid to identify for themselves the location of the gecko in the neighborhood around the school.

### Strands and Expectations

**Strand:** Measurement and Geometry

**Overall Expectation:** MGV .03B

**Specific Expectations:** MG 3.01, MG 3.02, MG 3.03, MG 3.04, MG 3.05, AG 3.04, NA 3.05

### Planning Notes

A map of the area around the school should be created by the teacher or an aerial map of your school's community may be downloaded from the website www.terraserver.microsoft.com. This should include the school location, school yard landmarks as well as a small shopping mall and some

housing. An example is included that can be used with this activity rather than the teacher create one of their own area. The map is placed on a set of coordinate axes so that the location of the gecko's cage in the science room is at the origin and significant landmarks are identifiable points on the grid, labeled A, B, C, D, etc.

The culminating task for students will be to create a dynamic geometry script to demonstrate that the clues in their ransom note produce the required location of the hidden gecko. If dynamic geometry software was not used in Unit 3, the students will complete this activity using paper and pencil strategies.

## Prior Knowledge Required

Concepts from Unit 3, comfort using dynamic geometry software

## Teaching/Learning Strategies

**Student Activity:** The students should be given **just** the co-ordinate axes grid with the significant points labeled as A, B, C, etc. The students will be told the co-ordinates of the location of the gecko. The students are to create a ransom note to be sent to the detective giving instructions how to get from the origin (in the school) to the location of the gecko (a given ordered pair). In this ransom note, their instructions must include at least 4 of the following:

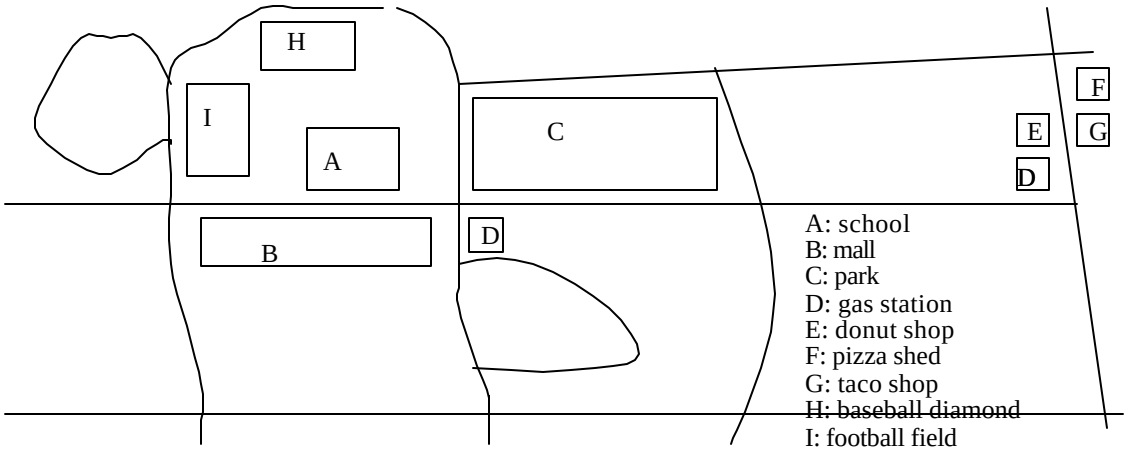
- use of the Pythagorean theorem
- using the equation of a line
- form a triangle using 3 significant points, construct the medians and use the medians to provide a clue
- form a triangle using 3 significant points, construct the altitudes and use the altitudes to provide a clue
- constructing an angle bisector
- constructing a perpendicular bisector
- finding a midpoint
- using properties of sides and/or diagonals of a polygon

- using the measures of interior and exterior angles of a triangle

Students will use dynamic geometry software to create a script which plays out the clues (or paper and pencil) to demonstrate that the clues in their note will lead the detective to the gecko. Once they have accomplished the task above, they can be given the map overlay. With the overlay in place, the co-ordinates students were originally given for the gecko will now tell them where the gecko is hidden in the community. They may also receive more information about the location, i.e. who lives there or who works there, etc.

**Teacher Facilitation:** An alternative to the teacher creating the scenario and the map would be to have the students create their own maps and decide where the gecko is hidden. Thus they will be working with their own scenario and be creating different suspect profiles.

The teacher will need to circulate as students work on this activity so that they can observe and gather informal assessment data. A checklist for observation would be helpful outlining what to look for. The teacher should also observe which members in the group are leading the discussion and which students need the assistance of the group to proceed. It is important to prompt when necessary and use different strategies to prompt, such as bringing someone from




one group to another group. The teacher can verify solutions by having groups signal when the geometry construction is complete and then demonstrating the construction on the computer screen.

A sample ransom note might contain the clues:

1. Draw a line through point A with a slope of 2/3.

2. This line should meet one of the points given.

3. Use this point and the points G and D to create a triangle.

4. Construct an altitude in this triangle through your point from #2.

5. Extend this altitude beyond the triangle. It should go through one of the points given.

6. Use this point and points A and E to create a triangle.

7. Construct a median through your point from #5. Extend this median below the triangle. The point it goes through is where the gecko is hidden.

The script that would be associated with this set of clues would look like:

Given: Point A, Point H, Point G, Point D, Point C, Point E

Steps: 1. Let [L]=Point (3.00,2.00) on the rectangular grid. 2. Let [ac]=Line between Point [L] and Point A. 3. Let [ad]=Segment between Point H and Point G. 4. Let [ae]=Segment between Point G and Point D. 5. Let [af]=Segment between Point D and Point H. 6. Let [ag]=Perpendicular to Segment [ae] through Point H. 7. Let [ah]=Segment between Point C and Point A. 8. Let [ai]=Segment between Point A and Point E. 9. Let [aj]=Segment between Point E and Point C. 10. Let[M]=Midpoint of Segment [ai]. 11. Let [ak]=Line between Midpoint [M] and Point C. 12. Select [ak].

### Assessment/Evaluation Techniques

The assessment of this activity will include knowledge of the geometric properties, use of the dynamic geometry software (or other construction tools), and communication using proper terminology in their clues and the use of appropriate language of mathematics to clearly direct the detective to the gecko.

## Activity 5 - Creating Their Own Clues

(This is an optional extension for students who have additional time or want to work on their own time to expand their suspect's profile.)

## Time: 2 hours

## Description

This activity is an extension and could be pursued by students who have readily completed the other clues or by students who want to explore an idea on their own time. Students will pose their own clue(s) using their knowledge from the first three units.

## Strands and Expectations

**Overall and Specific:** All are possible.

## Planning Notes

Students will create "Exhibit E" using some of the skills developed in the first three units. They should try to use several skill(s) not addressed in any of the other activities if possible. In their written submission they need to include a possible solution to their problem.

The teacher will conference with the students as they are working through their activity to determine the appropriateness of their choice. Creativity in their use of mathematics should be encouraged. A possible suggestion would be that each student in the group will be responsible for one component of the activity. The teacher may decide to use some of the students' ideas in subsequent years.

## Prior Knowledge Required

Concepts from the first three units

## Teaching/Learning Strategies

Students will work in groups and submit individual written solutions (could be for their part only).

## Assessment/Evaluation Techniques

Teacher could assess Learning Skills through observations and conferencing. The rubric found in Unit 4 Appendix could be used to assess Knowledge/Skills in the written submissions.

## Activity 6 - Class Presentations of Suspects' Profiles
## Time: 3 hours

### Description

In this activity, students will make their presentations to the class. Their actual profile may take many different forms. Their presentation must provide a convincing argument of how they used the Exhibits A-D (E) to determine their profile.

### Strands and Expectations

**Strands:** All.

**Overall Expectations:** NAV .01; REV .01; REV .02; REV .03; AGV .02; AGV .03; MGV .02; MGV .03

### Activity Instructions

### Planning Notes

- Teachers will need to conference with students before the presentations begin. The students should know what is expected of them before the conference interview. A checklist might be collaboratively developed with the class beforehand. The purpose of the conference is to help the students prepare for their upcoming presentations and also to give the teacher some informal assessment data. All student groups should submit the final profile of the suspect (wanted poster, skit, radio announcement, etc.) on "Conference Day"; presentation order can be drawn by lot. This way all groups will be on a equal footing when presenting their profiles and there will be no enhancements as groups gather ideas from the first presenters.

- A peer assessment form will need to be created so that students can assess one another's presentations.

### Prior Knowledge

Presentation skills, use of peer assessment.

### Teaching/Learning Strategies

**Student Activity:** Each group will have about 15 minutes to give their presentation. Each student should play a role in the presentation, perhaps be responsible for one piece of evidence and explain how that piece of evidence fits into the overall suspect profile. Students will be assessing their peers when not presenting.

> **Teacher Facilitation:** The teacher will need to control the timing, have equipment available if needed.

### Assessment/Evaluation Techniques

The assessment of the presentations can be done by both the teacher and the class through peer assessment. To facilitate the peer assessment, the teacher should discuss assessment criteria with the students and together develop a peer-assessment checklist or rubric that emphasizes communication and application of knowledge. Some assessment criteria might include: clarity of ideas, organization, using math to make an argument, and using appropriate terminology.

The teacher assessment should be through the use of clearly defined rubrics, designed to assess student's knowledge, communication and application, as modelled in the rubrics found in Unit 4 Appendix. During the presentations, teacher could place the students' initials in the appropriate "block" in the rubric for ease of marking.

## Activity 7 - Sample Questions for a Final Exam

### Time: 1.5 hours

### Description

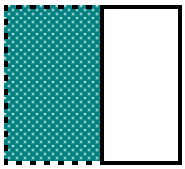
This paper and pencil test will include course expectations that may not have been addressed in the previous 6 activities. Sample pencil and paper questions have been provided. Teachers will add their own skill and knowledge -based questions to a selection of these sorts of questions to create a 1.5 hour exam.

### Strands and Expectations

**Number Sense and Algebra:** NAV .02, NAV .03
**Relationships:** REV .03
**Analytic Geometry:** AGV .01, AGV .03
**Measurement and Geometry:** MGV .01

### Planning Notes

The creation of this test should take into account all levels of achievement. It is new for many teachers to create questions that will provide opportunities for students to demonstrate a range of performances. The following are some examples. It would be reasonable to include several of these types of questions on a test:



(i) Calculate the maximum area for a given perimeter under the following circumstances. The police will need to secure an area outside a building where a crime took place. If they want to rope off the area outside the back of the building using 25m of yellow "DO NOT ENTER" tape, what are the dimensions of the largest rectangular area they can secure (or simply, rope off the largest area, not limited to a rectangular shape)?

(ii) Mobile Phone Charges: FON-mobile is a newly launched mobile phone provider who will offer low rates to attract customers away from its competitors. Customers can choose from two different plans:

FON-Mobile Plan A - $15 rental per month and a call charge of $0.30 per minute for air time.

FON-Mobile Plan B- $30 rental per month and a call charge of $0.10 per minute for air time.

Write a brief report explaining which scheme will be best for different customers, based upon the number of minutes that they might be on-line in a month. Your report should indicate when Plan A is better and when Plan B is better. Justify your responses.

(iii) The residents who live near a school call the police to report a speeding motorcyclist. They notify the police who immediately set out in pursuit. The motorcyclist's distances, $d$, traveled in km after a time, $t$, in minutes are given by the following equations. Consider time zero as the moment the police learn of the speeding motorcyclist.

Motorcyclist: $d = 2t + 3$

Police: $d = 2.5t$

Using a method of your choice answer the following questions. Show your work.

(a) How far ahead of the police is the motorcyclist at the moment the police begins the chase? Explain your answer.

(b) How long does it take the officer to catch the speeder? Explain your answer.

(c) In the time it takes the police to catch the speeder, how far has each of them traveled?

(d) Determine each vehicle's speed in km/min and km/h.

(e) If the motorcyclist was traveling at 135 km/h how long would it take the police to catch him?

(f) How fast would the police need to travel to catch the motorcyclist 6 minutes sooner?

(g) With this problem constant speed for the police car was assumed. However, upon learning of the motorcyclist, the police would need to accelerate to catch the speeder, Sketch the graphs of the 2 crafts with this in mind.

(iv) The function $y_1 = 4t + 3$, $t \geq 0$, $t$ in seconds, $y_1$ in meters describes the position of a runner. Distance is measured from the starting line and we start looking at $t = 0$.

(a) Where is the runner in relation to the starting line when we start looking? Explain your answer.

(b) How fast is this runner moving?

(c) A second runner's position is given by the function $y_2 = at + b$, $t \geq 0$, $t$ in seconds, $y_2$ in meters. What has to be true about the values of $a$ and $b$ if the second runner is at the starting line at the instant we start looking, and catches the first one at some later time?

(v) Given the two job offers below, determine the *better-paying* summer job. Explain your reasoning.

*Offer 1:* At Timmy's Tacos you will earn $4.50 an hour. However, you will be required to purchase a uniform for $45.00. You will be expected to work 20 hours each week.

*Offer 2:* At Kelly's Car Wash you will earn $3.50 an hour. No special attire is required. You must agree to work 20 hours each week.

(vi) Chris' Cheezy Pizza charges $5 for a 12-inch pepperoni pizza and $5 for delivery. Myrna's Meaty Pizza delivers for free, but charges $7 for her 12-inch pizza special. If you were ordering Pizza for the school year-end party, which company would you order from and why?

(vii) Once the suspect in a crime was uncovered rumours started to fly about "Who Dun It." As soon as one person found out who it was, they told 3 people within 5 minutes.

(a) If for each person who found out they told 3 more people within 5 minutes, how long would it take your whole school population to find out who the suspect was (Assume there are 1500 people in the school)? Show a full solution to justify your answer.

(b) How much longer would it take the rumour to spread if only 2 people were told in by each new person hearing the rumour?

## Teaching/Learning Strategies

Students should be adequately prepared for this test by completing the review materials and sought the teacher's help on content with which they had difficulty.

## Assessment/Evaluation Techniques

Assessment of most of this test will follow traditional practice. To assess the open-ended questions, use a rubric for problem solving and communication, modelled after the rubrics found in Unit 4 Appendix.

# Appendix

## Unit 4 - A Rubric for Observing Students at Work

**Overall Expectation:** **REV.01 - Determine relationships between two variables by collecting and analyzing data**

**REV.03 - Describe the connections between various representations of relations**

| Categories | 50-59% (Level 1) | 60-69% (Level 2) | 70-79% (Level 3) | 80-100% (Level 4) |
|---|---|---|---|---|
| **Thinking/Inquiry/**<br><br>**Problem Solving** | | | | |
| • collects, organizes and analyzes data to determine relationships between variables | • collects, organizes and analyzes data under direction from group | • collects, organizes and analyzes data with some help from the group | • collects, organizes and analyzes data independently | • collects, organizes and analyzes data effectively and independently |
| | • accepts with limited understanding arguments in choice of methods for collection, organization and analysis and has difficulty making inferences | • understands and follows arguments in choice of methods for collection, organization and analysis of data and make inferences | • uses arguments in choice of methods for collection, organization and analysis of data and making inferences with some minor flaws | • uses convincing arguments in choice of methods for collection, organization and analysis of data and making inferences |

| Categories | 50-59% (Level 1) | 60-69% (Level 2) | 70-79% (Level 3) | 80-100% (Level 4) |
|---|---|---|---|---|
| **Communication**<br><br>• communicates findings of an experiment using appropriate mathematical forms | • communicates minimal explanation of the solution with limited or no use of mathematical terminology | • communicates vaguely using incomplete justification and often uses inappropriate mathematical terminology | • communicates frequently using correct and appropriate mathematical terminology and some justification | • communicates clearly and consistently using correct and appropriate mathematical terminology and justification. |

## Unit 4 - A Rubric for Assessing Student Submissions

**Overall: REV.02B - Compare the graphs and formulas of linear relations; REV.03B - Describe the connections between various representations of relations**

| Categories | 50-59% (Level 1) | 60-69% (Level 2) | 70-79% (Level 3) | 80-100% (Level 4) |
|---|---|---|---|---|
| **Knowledge/ Understanding &Its Use In Applications** | | | | |
| • construct tables of values, scatterplots, graphs, lines of best fit<br>• determines relationships of variables, and applies them to solve the problem | • constructs tables of values and scatter plots and applies them with limited understanding (needs assistance from group, teacher, unit 1 notes)<br>• determines line of best fit and applies to this context with much assistance (group, teacher, notes)<br>• formulates a hypothesis with assistance and limited understanding of the relationship | • constructs tables of values and scatter plots and applies them to this context with inconsistent accuracy<br>• determines line of best fit and applies it to this context with some assistance<br>• formulates a hypothesis and determines the relationships with assistance | • constructs tables of values and scatter plots using appropriate techniques and applies them to this context<br>• determines line of best fit and applies it appropriately in this context<br>• formulates a hypothesis that shows a general understanding of the relationship and uses it to estimate height of suspect | • constructs tables of values and scatter plots with ease and systematically applies them to this context<br>• determines equation line of best fit and applies it appropriately and easily to this context<br>• formulates a hypothesis and makes predictions that show complete understanding of the relationship |