Public District School Board Writing Partnership

Course Profile

Principles of Mathematics

Grade 9
Academic

· *for teachers by teachers*

Acknowledgments

Public District School Board Writing Teams – Principles of Mathematics

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Ontario Mathematics Co-ordinators Association (OMCA)

Unit 2: Algebraic Models and Rates of Change

Time: 30 hours

Unit Description

Students use linear equations with variables *x* and *y* to algebraically summarize patterns derived from real-life contexts and to communicate using graphing technology. Working from real-life contexts, students develop a "common-sense" understanding of slope as unit rate of change prior to the algebraic definitions. They also explore connections between initial conditions and the *y*-intercepts of lines. The intent is for students to understand slope, equation, and line concepts in a manner which lends itself to application when problem solving. Properties and equations of lines are investigated and algebraic manipulations are taught and practised as needed.

Strand(s) and Expectations

Some specific expectations from the Number Sense and Algebra, and Relationships Strands have been combined with overall expectations from the Analytic Geometry Strand. Weaving together the expectations of the strands in this way helps students make connections.

Analytic Geometry Strand Specific Expectations: AG1.01, 02, 03, 04; AG2.01, 02, 03, 04, 05; AG3.01, 02, 03, 04, 05, 06, 07, 08.
Number Sense and Algebra Specific Expectations: NA1.01, 02, 03, 04, 05, 06; NA2.06; NA3.01, 02, 03, 04, 05, 06; NA 4.01, 02, 03.
Relationships: RE1.01, 03, 04, 05, 06, 07; RE2.01, 03; RE3.01, 02, 04.

Activity Titles

What follows is a suggested sequence for teaching Unit 2. The timing for activities and skill development is included. This Profile develops the mathematics in a sequence that may be different from the sequence used in previous courses of study, and weaves expectations from all strands together.
Since there are specific times when it would be best to introduce certain vocabulary and notation, and to develop certain skills, the outline for Unit 2 details when specific algebraic skills are required. Some of the activities include a large amount of skill development as indicated in the [square brackets]; other activities may require additional time for skills identified as Follow-Up Skills. Time has been allotted, in the table below, for the skill development within or following each activity. There is an additional 150 minutes of asterisked * time in this unit for skill building needs, as identified by the teacher.

Activity 2.1 & Follow-Up	**Match Me Up!** [$y = mx$ in contexts, where m is unit rate. No x's and y's until algebraic models are summarized. Then, use x and y notation so that graphing calculators can be instructed to create graphical models using "$y =$" form.] Follow-Up Skills: plot points in all 4 quadrants; use an equation in "$y =$" form to create a table of values to plot on the Cartesian plane	150 minutes
Activity 2.2 & Follow-Up	**Ramps 'R Us** [slope $=\frac{\text{rise}}{\text{run}}$] Follow-Up Skills: reviewing Pythagorean theorem; solving equations from knowing 2 of the variables in slope = $\frac{\text{rise}}{\text{run}}$	150 minutes
Activity 2.3 & Follow-Up	**Slippery Slope** [slope = $\frac{\Delta y}{\Delta x}$ developed through motion of students; slope = $\frac{y_2 - y_1}{x_2 - x_1}$ from contexts and tables; numeracy skills] Follow-up Skills, as homework: graphing lines, given slope and a point	75 minutes
Activity 2.4 & Follow-Up	**Programs for Sale!** [Use $y = mx + b$ form when (x_1, y_1) is a point on the y-axis and represents initial conditions in a partial variation context; solve simple linear equations by inspection; use list features of graphing calculators to enter formulas, as with spreadsheets] Follow-Up Skills: combining like terms; using distributive property; common factoring; exponent laws; solving linear equations involving some algebraic manipulation; reading and interpreting intersections of lines from graphs	300 minutes
Activity 2.5	**What's My Spring? Stretching a Penny** [Apply $y = mx + b$ form in a context that yields somewhat messy data; form and solve equations]	75 minutes
Activity 2.6 & Follow-Up	**Sunshine, Whiskers, and Windmill** [Investigate slopes of parallel and perpendicular lines and lines that are reflections in either the x-axis or y-axis] Follow-Up Skills: graphing lines, given slope and a special point, the y-intercept; forming the equation of a line, given slope and y-intercept; forming the equations of families of lines that share a slope, or a y-intercept	150 minutes
Activity 2.7 & Follow-Up	**Break the Bank!** [Re-arrange from $Ax + By + C = 0$ to $y = mx + b$ form to graph; determine the x- and y-intercepts of a line, given $Ax + By + C = 0$ or $y = mx + b$ forms; solve linear equations] Follow-Up Skills: rearrange linear equations; graph lines, given the intercepts, any two properties of the line, or the equation in any form	225 minutes

Activity 2.8 & Follow-Up	**Fireworks and Twinkle, Twinkle** [Develop $y - y_1 = m(x - x_1)$ and $y = y_0 + m(x - x_0)$ forms of the equation of a line] Follow-up Skills: graphing lines, given slope and a point, or given two points; forming equations of lines having properties like those of another line (e.g., parallel to one line and having the same *y*-intercept as another)	225 minutes
Activity 2.9 & Follow-Up	**All in the Family** [Graph lines and curves from equations] Follow-Up Skills: recognize linear vs non-linear relations from tables of values and equations; numeracy skills; graph $y = b$ and $x = a$	75 minutes
*** Time to practise skills**		150 minutes
Activity 2.10	**Planning for a Trip: a Summative Assessment Activity** Sample questions for a pencil & paper test: Confirm or Deny; Roof Trusses	225 minutes

Prior Knowledge Required
Unit 1

Unit Planning Notes
- The first activity is intended to introduce the use of *x* and *y* notation, with *x*'s representing independent variables, and *y*'s representing the dependent variables. Until now, letters having meaning in specific contexts have been used. To compare or summarize relations, or to instruct graphing technology to draw a graph, *x*'s and *y*'s are used.
- The word "slope" is introduced as a measure of inclination of a line through the context of wheelchair ramps. At this time, rates of change are connected to the abstraction of slopes of lines.
- The summative assessment activity in Unit 2 is intended to help prepare students for the type of activities in Unit 4.

Teaching/Learning Strategies
Small group organization of students works well as the comparisons of equation and graphical models for relationships are explored. Issue each student a graphing calculator, but arrange students in pairs as they learn new techniques so that they can help each other. Independent work is important in developing skills with algebraic manipulation and graphing calculators.

Using graphing calculators to reproduce Kitty's Whiskers, Rays of Sunshine, and other designs using sets of lines creates an interesting context for practice with equations of lines in $y = mx + b$ and $y - y_1 = m(x - x_1)$ forms. Games like 'Battleship' make the learning of co-ordinate graphing fun.

Direct teaching of algebraic manipulation skills can be moved from concrete to pictorial to abstract symbolic stages through the use of algebra tiles. Time has been allotted for this in the Follow-up skills part of many activities.

Assessment/Evaluation Techniques
As in Unit 1, a variety of assessment tools and strategies is recommended. Performance assessments may be used to effectively assess Thinking/Inquiry/Problem Solving, Communication, and Application categories of the Achievement Chart when students do open-ended tasks. Learning Skills can be assessed using teacher and peer observation, and self-reflection. Rubrics and rating scales are useful when a wide

range of performance is expected and when many complex criteria are to be judged. Whereas, checklists and marking schemes can still be used for more traditional tasks with predictable solutions.

Resources

Visualized Geometry: A van Hiele Level Approach. Portland, Maine: J. Weston Walch, 1990.

www.kings.k12.ca.us/math/lessons/ti83tutorial.html for instructions and sample data for using the LIST features of the TI-83+

Classroom Activities & Teacher Resources from Texas Instruments
www.ti.com/calc/docs/activities.htm

Ramp criteria
http://calder.med.miami.edu/pointis/ramp.html

Activity 1: Match Me Up!

Time: 150 minutes

Description

Students construct tables of values for direct variation from a variety of scenarios. Lines of best fit are drawn by hand and an equation for each line is developed. Students recognize that the multiplier or co-efficient in the equation relates to steepness, that several scenarios could produce lines with the same steepness, and that algebraic model can represent a variety of situations. The students then replace the independent and dependent variables in their equations with x and y to enable them to graph their relations with a graphing calculator. The follow-up graphing skills include introducing the four quadrants of the Cartesian plane and graphing equations of the form "$y =$" without technology by creating tables of values.

Strand(s) and Expectations

Strand(s): Number Sense and Algebra, Relationships, Analytic Geometry

Specific Expectations: NA1.01, .03, .04; RE1.04, 05, 06; RE2.01, 03, AG2.02, .03, .04, AG3.01, .02, .03, 07.

Planning Notes

- The teacher must have identical, quadrant I grids photocopied onto acetate sheets for each of the groups. See the Appendix for a template. A washable overhead marker is required for each group of three students.
- A photocopy or overhead of the Class Example is needed so that students see the questions that need to be answered in their groups.
- A photocopy of the scenarios should be made and cut into strips so that each group receives one scenario on a strip of paper.
- Lead a full class summary of the results from groups, drawing out the comparisons between the graphs, and introducing the usefulness of x and y notation.
- Provide a class set of graphing calculators, an overhead graphing calculator and LCD panel, and grid paper.

Prior Learning Required

From Unit 1: working with integers and rationals; discrete vs connected points; dependent vs independent variables; first differences; determining trends and patterns by making inferences from graphs; identifying and discussing patterns in algebraic terms; substituting into and simplifying an expression; entering lists and graphing scatter plots using the graphing calculator.

Teaching/Learning Strategies

This activity is broken into two parts. The first part begins with the teacher working through the scenario below with the full class. The students then work in groups, examining different scenarios that are similar to the worked scenario. Graphs are then compared and generalized which leads into the introduction of graphing calculators to graph equations using x's and y's. The second part of the activity begins with a full class exploration of using the graphing calculators to graph lines in the form of "y =". This includes moving from data and lines in the first quadrant to relationships in all four quadrants.

Part I

Teacher Facilitation: The teacher introduces a scenario like the one below, and outlines the questions to be considered in later scenarios. This example serves to review the concepts from Unit 1 that are needed for the Activity that follows. It is important to review with students how to determine dependent and independent variables using clues like unit rates. In expressing relations as equations, students should be using meaningful variables. DO NOT use x's and y's yet.

Class Example

A crystal growing kit contains enough material to perform two experiments. Complete the following steps to explore the relationship between the number kits and number of experiments.

i) The number of experiments performed depends on the number of kits purchased. Construct a table of values that shows the number of experiments that can be performed if up to five separate kits are purchased.
ii) Add a column in your table for first differences for the number of experiments that can be performed
iii) What is the independent variable? The dependent variable? How did you decide?
iv) Should the points be connected or not? Why?
v) Construct a scatter plot by hand.
vi) Write an equation that would describe this relation using letters appropriate for your scenario.
vii) What units are associated with the number in your equation? What does this number represent?
viii) Describe the connection between your equation and the first differences.

Teacher Facilitation: Place the students in groups of three and provide each group with a scenario and an acetate for constructing the graph of their scenario. The acetate should have a set of horizontal and vertical axes with appropriate scales photocopied onto it in order to facilitate the comparison of graphs later on. Students carry out the eight steps outlined in the previous example for the scenario that they are given.

The teacher should observe groups as they work to make sure that dependent and independent variables are being identified correctly and that correct decisions are being made regarding graphing of discrete and continuous data.

Student Activity

Scenario 1: A radio-controlled model car travels at a speed of 2 m/s. Graph the relationship between time (in seconds) and distance (in metres).

Scenario 2: Luke is purchasing packages of peanut butter cups to sell at a school dance. Complete the table of values for the numbers of packages given. Graph the relationship between the number of peanut butter cups and the number of packages between 0 and 50.

Number of packages	10	20	30	40	50
Number of peanut butter cups	30				

Scenario 3: In the tricycle department of a toy store, the number of wheels depends on the number of tricycles. Graph the relationship between the number of wheels and the number of tricycles.

Scenario 4: Bob's sock drawer is a mess. All of his socks are in it, but none of them have been put together in matched pairs. The number of pairs of socks depends on how many socks are in the drawer. Graph the relationship between the number of pairs and the number of socks.

Scenario 5: Photocopying on both sides of a piece of paper saves money and space. The number of pieces of paper required for a copy of a document depends on the number of pages to be photocopied. Graph the relationship between the number pieces of paper required and the number of pages being copied.

Scenario 6: Chocolate bars are often sold in packages of two. The number of bars you have depends on the number of packages you buy. Graph the relationship between the number of chocolate bars and the number of packages purchased.

Scenario 7: The number of participants in a chess tournament depends on the number of chessboards available. Graph the relationship between the number of chessboards and number of participants.

Scenario 8: Legal documents are often produced in triplicate. The number of copies depends on the number of documents prepared. Graph the relationship between the total number of copies and the number of documents prepared if the documents are produced in triplicate.

Teacher Facilitation: Once the groups have followed the eight steps and have their graphs on the acetates, the teacher brings closure to the activity by summarizing findings and connections with the full class. For example:

i) Ask a member of the group with Scenario 3 to bring their acetate to the overhead projector and explain the graph.
ii) Ask which other group has a graph that matches Scenario 3 in some way. (Scenarios 2 and 8 should match in steepness; Scenario 4 matches for disconnectedness)
iii) Focus on the steepness comparison. Ask students from group 2 and 8 to come up to the overhead and explain their graphs.
iv) Pile the three acetates on top of each other to show that they match identically for steepness.
v) Ask the following questions and draw out all of the mathematics:
    1. Why did the graphs match? (identical tables of values, same first differences, always tripling, same type of equation)
    2. What is different about the three graphs? (discreteness, title on graph, labels on axes, units on axes, units on the number in the equation (the unit rate))
    3. What is the real life meaning of the numerical multiplier in each equation?
vi) ***Introduce the idea that all three relationships could be summarized as $y = 3x$ where x represents the independent variable and y represents the dependent variable.***
vii) Repeat using Scenarios 1, 6, and 7, then Scenarios 4 and 5, introducing direct variation vocabulary, and *xy* notation.

Follow-up: Before starting, discuss the need to use the variables x and y when using a calculator, and then replace the independent variable with x and the dependent variable with y in the specific scenario that you are using.

Using a full class presentation, the teacher chooses one of the above Scenarios and coaches the students through the steps for constructing a scatter plot and graphing the equation of the line using the graphing calculators.

Students:

- clear all lists and previous graphs
- enter the data into two lists
- create a scatter plot using those two lists and view it using the ZOOM, ZoomStat feature

After examining the scatter plot, check the correctness of the equation that was determined for the relationship.
Does the line pass through the points of the scatter plot? Does the equation produce a fitting model?
The teacher needs to discuss the fact that we are representing discrete data with a continuous line. However, this is similar to talking about lines of best fit for discrete data.

Student Activity or Homework:
The students are given further scenarios of direct variation to graph and develop equations. These scenarios should include decreasing relations (e.g., the distance below sea level as a submarine descends at 5 m/s), relations that have fractional slopes (e.g., currency exchange rates) and relations that require different scales (e.g., distance driven vs gas consumed).
Ask students to describe a situation that would explain the events illustrated by an equation for a direct variation.
This would also be a good place to take some time to review skills regarding ratios and rates.

Part II

Teacher Facilitation: When students have completed the above activity or homework, the teacher should use three sets of data from their work to "take up" and extend the activity in a whole class setting using the overhead graphing calculator. Students can be working either in pairs or individually with graphing calculators. The class enters the table of values in the lists, creates a scatter plot and enters the "$y =$" equation for each of the three sets chosen by the teacher. The teacher should choose sets of data that include positive, negative, and fractional slopes and discuss: What is the same about these graphs? Different? What happens to the line when the multiplier or co-efficient is negative or positive?

Follow-up Graphing Skills:
After the discussion of the previous work concludes, the teacher should introduce students to graphs in four quadrants by showing students how to move from Zoom-stat to Zoom-standard. This leads into a formal discussion and labelling of the axes for the four quadrants including:

- Locating ordered pairs in each quadrant;
- Playing a short game of Battleship to familiarize students with plotting points in all quadrants;
- Graphing linear equations of the form "$y =$" by hand by creating a table of values. This helps students understand the process of creating a graph and reduce the "magic" of a calculator and understand different ways of modelling a relation. Use integer and fractional values in the equations and independent variables.
- Students may check the hand-drawn graphs by inputting the equations in the graphing calculator.

Assessment/Evaluation Techniques

While students work in pairs or groups, the teacher can gather data on Learning Skills such as teamwork, working independently, and work habits, using Appendix 1 (Phase 1). This activity also provides opportunities for formative assessment to determine areas where students need more assistance. This includes recognizing gaps in knowledge or understanding about relationships from Unit 1; difficulties communicating with appropriate terminology; or weak technology skills. This informal formative assessment guides the teacher in terms of the degree of review or prompting that is required.

Activity 2.2: Ramps 'R Us

Time: 150 minutes

Description

In this activity, students use the specifications for designing wheelchair ramps to investigate slope in the "rise over run" form.

Strand(s) and Expectations

Strand(s): Number Sense and Algebra, Analytic Geometry
Specific Expectations: NA1.03, .05, NA3.05, AG2.01, .05.

Planning Notes

- The teacher may wish to show a brief movie clip that shows a person using a wheelchair. (e.g., *Coming Home* - the scene where John Voight is learning how to navigate a ramp; *Forrest Gump* - the scene where Lt. Dan uses the ramp to his boat; or use slides of ramps from local surroundings.)
- Provide students with rulers and graph paper.

Prior Learning Required

Pythagorean theorem; ratios; converting between fractions, decimals and ratios; drawing to scale

Teaching/Learning Strategies

Teacher Facilitation: Teachers need to review the Pythagorean theorem, working with ratios, converting between fraction, decimals, and ratios, and the use of scale drawings before the activity or as the need arises. Students work in pairs for this activity.

Student Activity: Ramps 'R Us

If a family member becomes confined to a wheelchair, the home must be outfitted with ramps to provide access for that person.

Here are a few criteria adapted from suggestions by rehabilitation specialists (from http://calder.med.miami.edu/pointis/ramp.html):

- The maximum incline recommended for wheelchair users is 1:12, (i.e., for each centimetre in height, the ramp must extend 12 centimetres).
- For exterior ramps in climates where ice and snow are common, the incline should be more gradual, at 1:20.
- For unusually strong wheelchair users, for extra-powerful motorized chairs, and if the person is lightweight but the pusher is strong, the ramp can have an incline of 1:7. The steepest ramp should not have an incline exceeding 1:5.
- There should be at least 150 cm of straight clearance at the bottom of the ramp.

PART A

Refer to the descriptions of the following three clients as you design wheelchair ramps.

1. Client A lives in a split-level house. He owns a very powerful motorized chair. He wishes to build a ramp that leads from his sunken living room to his kitchen on the next level. The height of the ramp must be 60 cm.
2. Client B requires a ramp that leads from her back deck to a patio. She is of average strength and operates a manual wheel chair. The deck is 25 cm above the patio.
3. Client C lives in Sudbury where ice and snow are a factor. She is healthy, but not particularly strong. Her house is a single level bungalow but the front door is 0.45 m above ground level. The path that

leads directly to the front door from the street is 60 m long. Keep in mind that the ramp does not need to be 60 m long.

On a piece of graph paper design a ramp that would meet the criteria listed above and that would conserve ground space.

Construct a scale drawing for each design. Justify your reasoning for each design. Watch the units!

PART B

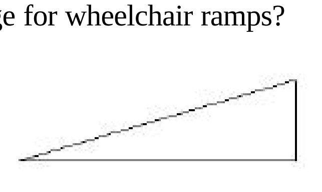
Mathematicians call the steepness of an incline slope. The slope can be calculated by dividing the height of a ramp (called rise) by the horizontal length (called run). This is often written as $m = \frac{\text{rise}}{\text{run}}$ where m is the variable used to identify slope.

- Determine the rise and run for each ramp from part A by forming a right angle triangle under each ramp. If your design includes resting points, use only one of the inclined sections for the chart.
- Sketch and label each ramp in the space provided.
- Complete the following table.

Wheelchair Ramps				
Ramp #	**Diagram**	**Rise, Vertical Distance (cm)**	**Run, Horizontal Distance (cm)**	**Slope, $m = \frac{\text{rise}}{\text{run}}$**

Questions

1. Compare the slopes of your ramps. If all of the ramps were of equal lengths, which incline would be the easiest one to push a wheelchair up? The most difficult?
2. The following is a scale drawing of a ramp. Measure the sides and determine the slope. Does this design fall within the acceptable range for wheelchair ramps?



3. How long is each of the ramps in your chart?
4. A building code requires a slope of 1:12 for a wheel chair ramp. If the length of a wheel chair ramp is 13 m and its horizontal distance is 12 m, is it safe?
5. What would be a good slope for a ski jump or skate board ramp? Explain your reasoning.
6. What is the slope of a rest platform? Explain.
7. What is the slope of a vertical wall? Explain.

Extension Question

Locate a wheel chair ramp in your community. Measure the rise and the run and calculate the slope. Does it fall within the suggested range?

Challenge Questions

1. A safe wheel chair ramp has a slope in the range from 1:5 to 1:20. If a ramp has a length of 20 m, what are the ranges of values for the rise and run of the ramp?
2. On a long ramp of any steepness or on a steep ramp, level rest platforms are needed every 3m of horizontal distance and should be 2 m long. Assume that such a ramp is a 2 m square. Given this information, how would you change your design for client C in Part A?

Teacher Facilitation: Bring closure to the Ramps inquiry by connecting steepness, developed in Unit 1- Activities 6 and 10, to slope. It might be advisable to re-visit some of the scenarios in Unit 1 and discuss them using slope vocabulary. Some of the questions in this activity provide beginning points for discussion surrounding such concepts as zero and undefined slopes.

Homework:

Students can be assigned other "rise over run" and Pythagorean theorem practice from their textbooks.

Equations resulting from knowing 2 of the variables in the formula $m = \frac{\text{rise}}{\text{run}}$ should also be practiced.

Assessment/Evaluation Techniques

The teacher may use this activity as an opportunity for informal, formative assessment of numeracy skills and facility with the Pythagorean theorem. This could be as simple as making observations to identify students who require remediation.

As this activity is inquiry-based, the teacher could assess students' problem-solving skills, such as risk-taking or testing out a variety of solutions, as they design ramps to satisfy the given criteria in ramp design.

A journal entry with students reflecting on other situations where differences in slope make a significant difference (roofs, highway ramps, skateboard ramps) may also be appropriate. This journal entry would help the teacher assess the students' application of knowledge and communication skills.

Activity 3: Slippery Slope

Time: 150 minutes

Description

Using the Slippery Slope Activity worksheet, students investigate slope as the change in y divided by the change in x. They observe and explain when and why the slope is positive or negative and compare the value of the slope of a line to its steepness and its direction. A follow-up exercise focusses on the calculation of slope using a formula, which sets the stage for determining the equation of a line given two points.

Strand(s) and Expectations

Strands: Analytic Geometry, Number Sense and Algebra, Relationships

Specific Expectations: AG2.01, .02, .05; AG3.01, .02; NA1.01, .02, .03; RE1.01, .02, .03, .04, .05, .06, .07; RE3.02.

Planning Notes

- Collect equipment needed: masking tape, metre sticks, graphing calculators, CBR™'s (Computer-Based Ranger), graphing calculator overhead display, class set of Slippery Slope Activity worksheets.

- Students work in groups of three to collect the data for the "Slippery Slope Activity." One student walks, one is responsible for the CBR™, and one is responsible for the calculator. They may change roles for different trials.
- Students need floor space for setting up this activity.

Prior Learning Required

Activities 1.8 and 1.9 (distance/time relationships, analysing and interpreting relationships), Activities 2.1 and 2.2 (using x and y notation, using $m = \frac{\text{rise}}{\text{run}}$)

Teaching/Learning Strategies

Teacher Facilitation: For the first activity, Slippery Slope, group work should be interspersed with whole class discussion as needed. The teacher may want to refer to an overhead of the chart for this activity. Once the instructions to the students have been given, circulate and help groups as necessary. The teacher should make sure that the students are familiar with the procedure for setting up the CBR™. Students collect data and record their observations using the chart provided.

Student Activity: Slippery Slope Activity

(Modified from "Slippery Slope", *Math and Science in Motion*, TI Inc.)

In this activity, you will create Distance-Time plots by moving in front of a CBR™, find slopes on the plots, and determine the formula for the slope of a line.

You will need the following materials: CBR™ unit, TI-83+ and calculator-to-CBR™ cable, metre stick, and masking tape

Collecting the Data:

1. Three students work together to collect the data. One is the walker, one controls the CBR™, and one controls the calculator. You change roles for subsequent trials.
2. Place the CBR™ on a table or desk so that the sensor is aimed at or above the walker's waist. The height can be adjusted using textbooks, if necessary
3. Put a masking tape marker on the floor at a distance of 0.5 m from the CBR™ and at a distance of 3.0 m.
4. The walker stands at the 0.5 m mark and prepares to move away from the CBR™ at a slow and steady rate. When the walker is ready the calculator person presses [ENTER] to begin the walk. The partner presses the Trigger button on the CBR™ to stop the recording when the walker passes the 3 m mark. The walker must try to keep a steady pace for the whole walk.

The plot should look like a *straight* line that rises gently to the right.

5. If your line is reasonably straight, sketch it in your notebook and label the graph, Trial 1, and go to question 6. If not, press [ENTER] select 3: REPEAT SAMPLE from the PLOT MENU, and try again.
6. Using the [▶] and [◀] keys, find and record the co-ordinates of two points on the line in the chart below. Choose points near the beginning and end, so that the line between them is the straight line that best models the plot.
7. Repeat steps 3-5 for the following motions, sketching your results for each in your notebook.
   Trial 2 - moderate, steady walking away from the CBR™
   Trial 3 - slow, steady walking towards the CBR™
   Trial 4 - moderate, steady walking towards the CBR™.

Observations:

Answer the following questions, recording your results in the chart.

1. Refer to the graphs in your notebook and compare the four plots in terms of steepness of the line (slope), and the direction of the line (is it going up or down, from left to right?). Enter this information into the chart.

   The D (delta) symbol is shorthand to represent change and is often used in physics and other disciplines. In general,

   D = final condition - initial condition

   Dy (delta *y*) and Dx (delta *x*) denote the vertical change and the horizontal change from starting point to end point.

2. Calculate the vertical change (Δy) by calculating the change in *y* from starting point to ending point ($y_{ending} - y_{starting}$). Enter this amount into the table.
3. Calculate the horizontal change (Δx) by calculating the difference in *x* from the starting point to ending point ($x_{ending} - x_{starting}$). Enter this amount into the table.
4. Calculate the slope by finding the $\frac{\text{vertical change}}{\text{horizontal change}}$.

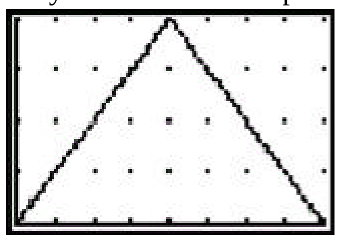
Collecting Data						Calculations		
Trial	**Starting Distance**	**Type of Motion**	**Steepness and Direction of Line**	**Starting Point**	**Ending Point**	**Vertical Change Δy**	**Horizontal Change Δx**	**Slope (m/s) $\frac{\Delta y}{\Delta x}$**
1	0.5 m	Slow, steady away from CBR™						
2	0.5 m	Moderate, steady away from CBR™						
3	3.0 m	Slow, steady toward CBR™						
4	3.0 m	Moderate, steady toward CBR™						

Questions:

1. What pattern do you notice in the slope values in the trials where you were walking *away* from the CBR™? *Towards* the CBR™?
2. What pattern do you notice relating the steepness of the lines and the speed that you walked away/towards the CBR™?
3. What pattern do you notice relating the steepness of the line with the values of the slopes?
4. What pattern do you notice relating the direction of the line with the slope values?
5. Hypothesize the slope of the line and draw a sketch for the following situations:
   (a) run away from the CBR™ (0.5 m - 3.0 m)
   (b) run toward the CBR™ (3.0 m - 0.5 m)
   (c) move to a point 2 m away from the motion detector, and then stop
6. Check your hypotheses using the CBR™.

Further Questions:

1. Look at the plots of the four trials. Come up with a general rule about where the graph starts on the vertical axis (distance axis).
2. What are the units for distance in this activity? For time?
3. What are the units for horizontal change? For vertical change?
4. Explain why the units for slope are in m/s.
5. Draw a sketch if a walker starts at 2.5 m away from the motion detector and walks at a rate of
   (a) 2 m in 3 sec;
   (b) -1 m in 2 sec.
6. Describe in words the motion indicated by this distance-time plot.



7. Create an interesting distance-time plot of your own and then describe in words the motion indicated by it.

Follow-Up Activity

Teacher Facilitation: Before beginning the follow-up activity, involve the class in a discussion of how to approximate and guess how the slope of a line segment will look. Students should be advised to consider both the steepness and the direction of the line segment. For example: ask the students to stand up so they can use their arms to approximate line segments. Ask them to imagine two points on the plane say (3, -4) and (6, 1) and then use their arms to estimate what the slope of the line segment might look like. Ask them to compare with two or three others and come to a consensus for how they should hold their arms. Repeat this process with other points that include line segments that have a variety of slopes from 0 to infinite in both directions. The ideas could be summarized in a chart that shows angles like 0°, 25°, 45°, 70°, and 90° so that students connect slope with the relative steepness or approximate angle.

Provide instructions for the pencil and paper activity below and introduce the representations of points using *x* and *y* with subscripts. Substituting into formulas might need to be reviewed, as well. The important idea here is to help students realize that the subscripts are used to designate coordinates of specific points; it doesn't matter which is the first point and which is the second and, when substituting into the formula, it is essential that students not mix up the coordinates of the points. Discuss the different forms that could be used for *m* values: rational numbers in $\frac{a}{b}$ or decimal form, if both are equivalent. Include a review of when the decimal forms of fractions are approximations as opposed to exact values.

Students may also need help determining how to use a point and the slope to extend the line segment in both directions.

Student Activity

1. Construct a grid on graph paper where the points on the *x*-axis and *y*-axis are laid out as follows:
   -10 #*x* # 10, -10 # *y* # 10
2. a) Plot the points A(-8, -9) and B(-3, -6). Join the points to form line segment AB.
   b) Construct a right angle triangle under the line segment AB.

c) Determine the rise and the run for the line segment.

d) Calculate and record the slope using $m = \frac{\text{rise}}{\text{run}}$.

3. Let A(-8, -9) be represented by (x_1, y_1) and B(-3, -6) be represented by (x_2, y_2).

a) Calculate and record the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

b) Now let B(-3, -6) be represented by (x_1, y_1) and A(-8, -9) be represented by (x_2, y_2). Calculate the slope using the formula.

c) Record your results in the table provided.

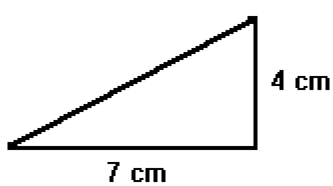
4. Repeat the process (steps 3b and c) for the other sets of points in the table.

Points	rise	run	$m = \frac{\text{rise}}{\text{run}}$	x_1	y_1	x_2	y_2	$m = \frac{y_2 - y_1}{x_2 - x_1}$
(2, 6), (-10, -4)								
(-10, -4), (2, 6)								
(-10, 5), (-6, 2)								
(-6, 2), (-10, 5)								

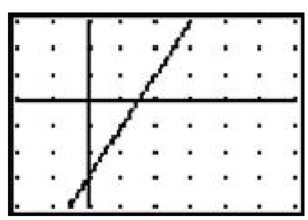
In your notes, answer the following questions:

1. Identify any differences between the slope when it is calculated using rise over run and when it is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Explain.
2. Identify any differences in the value of slope when point A is (x_1, y_1) and B is (x_2, y_2) and when the points are switched. Account for your observations.
3. For each pair of points, describe how you would start at one point and get to the other point using the slope. (Hint: Starting at the point (x_1, y_1) move up or down so many units and then left or right so many units to get to (x_2, y_2)).
4. Describe how you could find a new point on the line if your line segment needed to be extended.
5. You have now worked with three ways to calculate the slope of a line segment. Explain which method is the most convenient and use it to calculate the slope for each of the following:

a)



b)



c) Line through the points (-3, 1) and (4, -5)

Homework:

Include practice using the formula and extending lines using the slope and a point. Students could be assigned a journal entry asking them to relate the three methods of calculating the slope and describe situations or problems where they would use each form.

Assessment/Evaluation Techniques

The teacher could make observations on Learning Skills, such as teamwork and initiative, while the students are performing this activity.

If students submit their answers to the questions associated with this activity, then teachers could assess Communication Skills by focusing on: the clarity of the students' descriptions of the patterns that they recognized, and the use of appropriate mathematics terminology and appropriate units on axes. Reviewing students' journal entries also aids in the assessment of Communication.

A summative quiz could be given to ensure that students are able to calculate the slope of a line segment using various formulas; identify properties of line segments and graph line segments using the slope and a point.

Accommodations

- The teacher may have to lead parts of the instruction using a CBR™ and an overhead graphing calculator. Having students that benefit from kinesthetic activities model the activity may be helpful. Some individual instruction and prompting may be required.
- Some students may have difficulty describing how to graph line segments in both directions using a starting point and a slope. Some instruction/prompting may be necessary for some students.

Activity 2.4: Programs for Sale!

Time: 300 minutes

Description

In this activity, students develop an understanding of the formula $y = mx + b$ by making a connection between the initial conditions and the y-intercept, b. Instructions guide students to use the TI-83+ as a spreadsheet to generate tables of values, thereby offering an alternative way to answer questions where one of the variables is known. Points of intersection are found graphically and interpreted in context. As a follow-up, students develop their algebraic skills of collecting like terms, using the distributive property, and common factoring, and apply these new algebraic skills to solving linear equations. Students also learn to form the equation and graph the line whose slope and y-intercept are known.

Strand(s) and Expectations

Strand(s): Analytic Geometry, Number Sense and Algebra, Relationships

Specific Expectations: AG2.02, .04, .05; AG3.02, .03, .04, .05, .06; NA2.06; NA3.01, .02, .03, .04; NA4.01, .02, .03; RE3.01.

Planning Notes

- Graphing calculators are required for Activity 2.4.

Prior Learning Required

Students should know how to make a table of values for a linear relation from a description of a situation, graph from a table of values, compute slope of a line, form the equation of a linear relation, solve a linear equation by inspection or systematic trial, and use graphing technology to graph an equation in y = form.

Teaching/Learning Strategies

Teacher Facilitation: Students should be organized in pairs for this activity. The teacher intersperses students working in pairs with whole class discussion, as needed. Students should begin pair work on Programs for Sale! and Time out for Technology. Some class discussion should follow and then pairs should continue their work, moving to the activities: Ringaling, Reliable Appliance Repairs, and Decisions, Decisions.

Students are expected to write equations for each situation using *x* for the independent variable and *y* for the dependent variable. Intersperse pair work with whole class discussion and instruction. Some of these questions could be assigned as homework.
Students generate intersecting lines in Decisions, Decisions, Decisions! They are expected to find points of intersection graphically. Algebraic techniques are not expected until Grade 10.
Students are expected to use inspection or informal methods for solving the equations in the problems. Instructions on formal methods for solving equations should be scheduled after the worksheets are completed and before the 2.4 homework sheet.

Student Activity: Programs for Sale!

Carolyn gets a job at the Sky Dome selling programs. She is paid $10 per game and she is given 25 cents for every program she sells.

1. Create a table of values showing the pay she can expect for a game where the number of programs she sells varies from 0 to 100. Use increments of 10 programs in your table.
2. Calculate the first differences. What do they tell you about the relation?
3. What is the rate Carolyn is paid per program?
4. Write an equation.
5. Graph the relation. Should you connect the points? Why or why not?
6. What is the slope of your line? Compare your answer to #2 and #3?
7. Examine your table. What is her pay for selling 0 programs? Where does this information show on your graph?
8. A common method of writing equations of linear models is $y = mx + b$, where b is the initial value of the dependent variable and m is still ___________ (Fill the blank.)
9. How much would Carolyn be paid if she sold 45 programs? How could you use your graph to answer the question? Show how to use your equation to answer the question.
10. How many programs would Carolyn need to sell to earn $26.50 for a game? How would you use your graph to answer the question? Show how you would use your equation to answer the question.
11. What would be the equation if Carolyn were paid:
    a) $15 per game, plus 25 cents per program sold?
    b) $20 per game? $18 per game plus 30 cents per program?
    c) $15 per game and 50 cents per program?

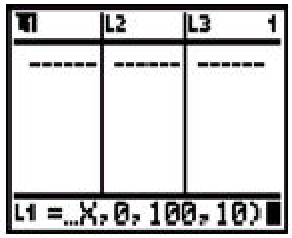
Student Activity: Time Out for Technology

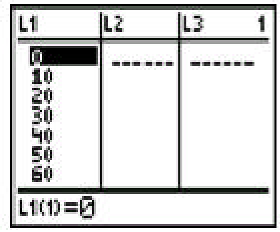
Graphing calculators can be programmed much like a spreadsheet on a computer to give up-datable tables of values. You can choose the increments for *x*. Follow the example below on your TI-83+.

The table of values you generated in Programs for Sale! could be done on a TI-83+ as follows:

a) Clear lists 1 and 2
   STAT
   1:Edit
   Up arrow to highlight L1
   CLEAR
   ENTER
   Repeat for L2

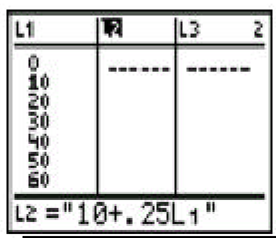
b) Enter the independent variable values in L1
STAT
1: Edit
Up arrow to highlight L1
LIST (or 2nd STAT)
Right arrow to OPS
Down arrow to 5:seq(
ENTER
X,X,0,100,10)
{This sets 0 as the lowest, 100 as the highest, and 10 as the increment for X}
ENTER

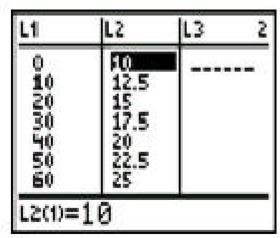




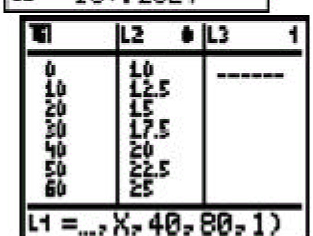
c) Enter the dependent variable values in L2
Right and Up arrows to highlight L2
“10+.25L1” {This
ENTER

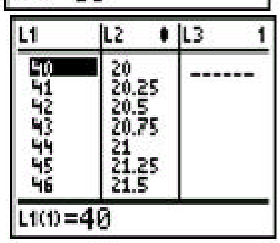
d) Changing values in L1 automatically updates L2 since you have put quotation marks around your formula for L2. You might want to try to do some updates to see how to use your calculator to answer questions 9 and 10. For example, if you guessed that the answers to 9 and 10 involved between 40 and 80 programs, change L1 as follows:
Left and Up arrows to highlight L1
CLEAR
LIST
Right arrow to OPS
Down arrow to 5:seq(
ENTER
X,X,40,80,1)
ENTER

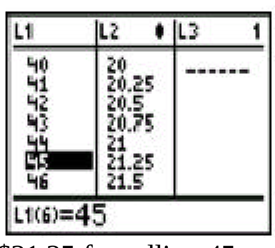


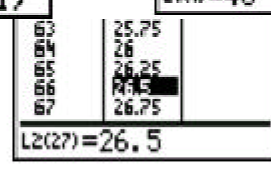


e) You can now use the Down arrow to scroll down the lists to read the amount Carolyn was paid for selling 45 programs, or how many she has to sell to earn $26.50.





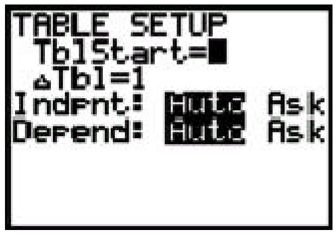




Carolyn is paid $21.25 for selling 45 programs.

Carolyn has to sell 66 programs to earn $26.50.

Alternatively, you can form the equation of a relation with *x* as the independent variable and graph it using Y =. You can read the lists that the calculator has generated by going to TABLE (2nd GRAPH). Use the arrow keys to scroll through the lists to read off values. The table starts at $x = 0$ and show increments of 1 unless you go into TBLSET (2nd WINDOW). On this screen you can set TblStart to start the table with whatever *x* value you want, and △Tbl to set whatever increment you want.



Student Activity: Ringaling!

A phone company charges $15 per month for an emergency cell phone service, plus a call charge of $1 per minute.

1. Use your TI-83+ to create a table of values showing the total charges for one month where the calls vary from 0 to 60 minutes.
2. Graph the relation.
3. Identify the slope and *y*-intercepts of your line. How do these relate to the phone charges?
4. Describe 3 different ways that you could find out how much the company would charge for a month when 45 minutes of calls are made.
5. Describe 3 different ways that you could find how many minutes were used for calls if the monthly charge was $43.
6. How would your graph be different if the company charged the minute rate for every full minute or portion of a minute used? [The graph you should have guessed is called a step graph. Do you see why?] Using this step model, what would the phone company charge you for a 30 second phone call? for 40 minutes and 2 seconds of calling during the month?

Student Activity: Reliable Appliance Repairs

The Reliable Repair Company charges $25 per visit plus $60 per hour for labour.

1. Use your TI-83+ to create a table of values showing the total charges for a repair that could take from no time at all to 8 hours.
2. Graph the relation.
3. Identify the slope and *y*-intercepts of your line. How do these relate to the repair charges?
4. How much would be charged for a repair that takes 3 hours and 20 minutes of labour?
5. If this company submitted an invoice for $160 for a repair at your home, how much time should have been spent on labour?
6. If the Reliable Repair Company changed their hourly rate to $65, but kept the fixed rate of a house call at $25, what would be the equation for their new charge structure? How would this change affect the graph?
7. If the Reliable Repair Company changed its fee per visit to $30 but kept their hourly rate at $60, how would the equation and graph compare to the original?

Student Activity: Decisions, Decisions, Decisions!

You are helping a friend decide which cellular phone package is best for her to buy. The cost of purchasing the phone is the same in all cases. Crystal Net charges a flat fee of $27 for up to 200 minutes per month, while Rover charges $12 per month, plus $0.15 per minute, and Ring Mobility charges $15 per month, plus $0.10 per minute.

1. Use your calculator to graph each relation, using different markings for each one.

2. For which time value are the costs the same for the Rover and Crystal Net options?
3. For which time value are the costs the same for the Rover and Ring Mobility options?
4. For which time values is Ring Mobility a better deal than both Rover and Crystal Net?
5. If your friend expects to use her cell phone between 40 and 100 minutes per month, explain to her which service you think she should choose.
6. If you were in charge of marketing for Ring Mobility and it was your goal to increase your market share, how could you adjust your pricing formula to attract customers from the other two companies? State the equation for your pricing structure. Explain the decisions you made in forming your equation, remembering that you have to make money for the company as well as attract as many customers as possible.

Follow-Up Skills Lessons:

Teacher Facilitation: Lead students to graphing equations of the form $y = mx + b$ without a table of values. Practice these.

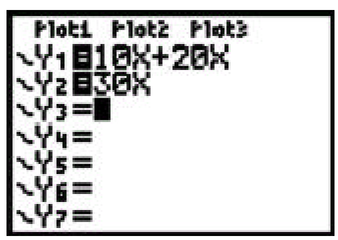
Use a situation like the following one to illustrate to students that $10x + 20x = 30x$ since the graph of $y = 10x + 20x$ and $y = 30x$ are the same.

Student Activity:

Conrad delivers 2 different newspapers to the same set of homes. His pay is 10 cents per paper from the Oakville Beaver and 20 cents per paper from the Oakville North News.

1. Using x to represent the number of homes and y to represent his pay, form equations to represent his weekly pay from each newspaper and his total combined pay.
2. Create tables of values for these 3 relations, including values from 0 to 40 homes.
3. What is the relationship between the pay scales for each of the two papers and the combined total? How can this be shown algebraically?
4. Enter your equations in a graphing calculator where $y_1 = 10x + 20x$ and $y_2 = 30x$.

Teacher Facilitation: Since these two lines are on top of each other, instruct the students as to how to get different markings for the two functions. Arrow to the left of y_2 and press enter 5 times until the circle option is displayed.



5. Compare the two displays.
6. Go into the table menu to view the values in the two tables. How do the values compare?

Teacher Facilitation: Lead students to conclude that $10x + 20x = 30x$ and explaining why this is so, within the context.

Use algebra tiles or Virtualtiles for concrete or visual manipulation to extend the algebraic skills of combining like terms. Students may well have used integers tiles in elementary school. To build on their use of colour and area from integer tiles, use the same two-colour distinction to differentiate between positives and negatives. To model x, use a tile that is x units long and 1 unit wide (the same unit size as used for each side of an integer square). To model x^2, use an x by x tile.

Introduce the distributive property and common factoring using an example like the following.

Student Activity:

A taxi charges a flat rate of $2 plus 16 cents per kilometre traveled.

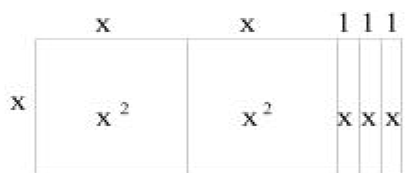
1. Construct a table of values for a single trip that range from 0 to 100 km.

2. Identify the slope and y-intercept, and use them to form the equation for this relation, using x as the distance traveled and y as the charge for the ride.
3. Suppose a person is using the taxi daily to get to and from work. Construct a table of values for their weekly charges (10 trips)
4. Identify the slope and y-intercept, and use them to form the equation for the relation in #3.
5. Realizing that your first equation represents the cost for 1 trip, what would you expect to have to determine the cost for 10 trips? Write an equation with this idea in mind.
6. Graph your relations in #2 and #5 on the same axe, allowing up to $200 in charges. How do your graphs compare?

Teacher Facilitation: Lead students to the realization that since the graphs of the relations $20 + 10 \text{ x } 0.16x = 10(2 + 0.16x)$ since the graphs of the relations $y = 20 + 10 \times 0.16x$ and $y = 10(2 + 0.16x)$ are identical.

Another situation that illustrates the distributive property or common factoring is: One pizza costs $6, plus $1 per topping. Find the cost of 4 pizzas if each pizza has *x* toppings. [4(6+1*x*) and 24+4*x* are the two ways that you could form the expression, so these must be equivalent] Introduce the concept that distribution and common factoring are inverse operations.

Use algebra tiles to further students' understanding of the algebraic operations of expanding [given the length times width form, find the area of the rectangular region], and common factoring [given the area of a rectangular region, find the length and width]. For example, this pictorial representation of algebra tiles shows that $x(2x + 3) = 2x^2 + 3x$. Tie this statement to the formula: length × width = area of a rectangle.



Teach students how to apply patterns to develop the exponent rules for multiplying and dividing monomials, then practise these rules. Solve linear equations involving integral and fractional coefficients. For this work, teachers should use discretion in choosing the upper level of difficulty they assign to specific groups of students.

Follow-up Homework: (Note that you may wish to use #7 as a pairs assessment task)

1. A salesman is paid $600 per week, plus 10% on sales.
   a) Create a table of values showing his pay in a week where his sales might vary from $0 to $2 000.
   b) Write an equation for the relation and sketch the graph.
   c) How much is the salesman paid if he makes $1 500 worth of sales?
   d) What sales need to be made to earn $850 in a week?
2. A waitress is hired at Lionel's Lunch Spot. She must pay $50 for a uniform (which is deducted from her pay) and she earns $6 per hour, before tips are added.
   a) Create a table of values showing her pay without tips for her first week, where she might work 0 to 40 hours.
   b) Write an equation and sketch a graph.
   c) How long would she have to work to earn $142 before tips?
   d) How many hours does she have to work to pay off her uniform? How is this information shown on your graph?

3. A car's gas tank holds 60 L of gasoline and the car can travel a total of 510 km on a full tank of gas.
   a) What does this information tell you about the graph of the relationship between fuel remaining and distance travelled? Draw the graph.
   b) How much gas is left in the tank when the car has travelled 100 km?
   c) How far has the car travelled when 38 L of gas remains?
   d) The "low level" warning light appears when there are 5 L of gas remaining in the tank. How much farther could the car go, once the warning light comes on, before running out of gas?
   e) What is the rate of fuel consumption in kilometres per litre? Write an equation which represents the amount of gas remaining in terms of the distance travelled.
   f) If the car gets a tune up, the fuel efficiency will increase to 9 kilometres per litre. How will this change the equation? How many more kilometres will the car be able to travel on a full tank of gas?
4. The cost to manufacture 200 baseball caps is $1 400. The cost for 500 is $2 600.
   a) Assuming that it is a linear relation, graph it.
   b) What is the initial cost if no hats are manufactured? Explain the answer.
   c) What is the cost for each hat added? How does this answer compare to the graph?
   d) Form an equation that expresses the cost in terms of the number of hats that are purchased.
5. Describe a situation that could be modelled by the following equation:
   a) $d = 2t + 3$, where d represents distance and t represents time
   b) $c = 50b + 100$, where c represents cost and b represents number of books
6. Ali is moving to Ottawa as a bicycle salesman. He is offered two methods of payment. Method A is a salary of $400 per week plus 5% commission on sales made. Method B is a straight commission of 15% on sales made.
   a) Form the equation for each method of payment.
   b) Graph both equations on the same grid.
   c) Under what conditions will the two methods of payment result in the same earnings? Explain how you arrived at your answer.
   d) What advice would you give to Ali about his choice on method of payment?
   e) If Ali could change his payment method on a monthly basis, what advice would you give him? Explain.
7. Create a situation of your own and the equation(s) which models it. Pose different types of questions. Show or explain how to answer the questions you posed.

   Teacher Facilitation: Ask the following types of questions as homework is taken up:
   i) How is the slope identified in each question? [the unit rate]
   ii) How is the *b* value identified in the situations? [the fixed initial quantity. Now is a good time to introduce the terms "*y*-intercept" and partial variation]
   iii) In what type of situation will the *b* be negative? Slope be negative?
   iv) When answering questions that require interpolation, which method do you prefer - using the equation, or looking at the graph, or reading from the lists on the calculator? Why?

Assessment/Evaluation Techniques

Question #7 in the Follow-Up Homework could be used as an in-class pairs assignment. The teacher could assess the Communication and Application Categories of the Achievement Chart by having students present orally or in written form.

As students work in pairs, the teacher could assess Learning Skills such as Team Work and Organization using rubrics provided earlier.

After this activity has been completed and concepts from Activities 2.1 to 2.4 consolidated, it would be appropriate to give a cumulative test on concepts learned through these activities. Questions on the test could cover the four categories of the Achievement Chart and assess the following expectations:

Knowledge:	Calculations of slope using a variety of slope definitions; identifying slope and *y*-intercepts in $y = mx + b$ form of the equation of a line; solution of equations
Thinking/Inquiry:	Justifying reasoning within the solution to a problem
Communication:	Describing the meaning of slope and *y*-intercept in context; describing situations that could be modeled by a linear equation; using appropriate mathematics symbols and terminology: slope, *y*-intercept, labeling axes, constant rate, fixed cost
Application:	Relating abstract concepts of slope and *y*-intercept to realistic contexts; recognizing the purpose and value of measures of slope in a variety of contexts (highway ramps, skateboard ramps, roofs)

Include questions that allow students to demonstrate Level 4 achievement of the expectations need to be included. Do this by asking extending questions and providing opportunities for students to explain their reasoning or show a more efficient or creative solution. Open-ended questions provide opportunities for students to demonstrate all four Categories at all four Levels of Performance.

Accommodations
The use of algebra tiles helps students who benefit from work at the concrete and pictorial stages before they abstract. They appeal to visual and kinesthetic learners. The use of Virtualtiles also helps visual learners.

Resources
Lesage, J., B. Scully, and J. Scully, *"Alge-Tile" Resource Binder*, Exclusive Educational Products.

Activity 2.5: What's My Spring? Stretching a Penny
Time: 75 minutes

Description
Students collect data using a CBR™, a slinky, and pennies (or similar experiment using springs or elastics and a ruler) to determine the relationship between the distance a spring is stretched and the mass (number of pennies) attached to it. Students use the slope/*y*-intercept form to determine the equation which models the data. They also determine the equations of graphs which represent other spring relationships and be expected to explain the physical meaning of the slopes and the *y*-intercepts in the contexts provided and to use their equations to answer other questions.

Strand(s) and Expectations
Strand(s): Analytic Geometry, Number Sense and Algebra, Relationships

Specific Expectations: AG2.04, .05; AG3.02, .03, .04, .05, .06, .07; NA3.02; NA4.01, .03; RE1.01, .02, .03, .04, .06, .07; RE2.02, .03; RE3.03.

Planning Notes
- For each group the teacher sets up a slinky suspended from the ceiling (or see your Science department for springs, retort stands, and clamps), a paper plate attached to it (to hold pennies), 30 pennies, a CBR™, and TI-83+ calculator to collect the data. See *CBR™ Explorations* book for more detailed information.

- The plates may need to be secured to the slinkies with ribbon and taped on (so that they do not slip off).
- A worksheet needs to be prepared to record their data, determine the equation of the line of best fit, give meaning to the slope and *y*-intercept in the given context and include extending questions. See Appendix 2.5A - Stretching a Penny Worksheet.
- Students who did this experiment in Activity 1.10 with a CBR™ could recall their data at this time and answer the questions from the work sheet.

Prior Learning Required

- How to determine the equation of a line by identifying the slope and *y*-intercept;
- How to use a graphing calculator to enter data into lists, graph an equation, and set appropriate parameters in the WINDOW.

Teaching/Learning Strategies

Student Activity

Students work in groups of three on the Stretching A Penny Worksheet (Appendix 2.5A), recording distances of the plate from the CBR™. They then answer the worksheet questions about the relation.

> **Teacher Facilitation:** To develop a context before starting the activity, the teacher could ask the students where the relationship between spring length and weight might be used. Some possible answers might include a bungie jump, a trampoline, or a mattress. Discuss possible sources of error and emphasize that the CBR™ should not record until the plate has stopped moving. Circulate around the room helping students as they collect their data and analyse it. Students may need to be reminded to find the slope and *y*-intercept as a way to find the equation. Data collected can be inserted into their calculators as lists. They could then verify the correctness of their equation by entering it into the calculator.
>
> **Note:** The equation is to be adjusted if it is not a fitting model for the data. Help may be needed with the sign of the slope.

Suggested Homework: Refer to Appendix 2.5B – What's My Spring? The first two grids show graphs of the length of a spring when a mass is attached to it. The third grid shows the graph of the length of a mattress spring when a mass is placed on it.

Follow-up: Appendix 2.5C applies student knowledge from the first two activities to the context of temperature.

> **Teacher Facilitation:** Note that the temperature intervals given in the charts are not equal to 1°. Students cannot use First Differences to calculate the slope. When the solutions are taken up from the Frozen Balloon activity, question #5 can be expanded further by bringing out the fact that according to the kinetic molecular theory, a sample of gas would exert zero pressure only when the molecules are not in motion. This would be the case when the temperature is at absolute zero. Absolute zero is the coldest possible temperature. The accepted value for absolute zero is -273.18°C.

Assessment/Evaluation Techniques

As students work in groups of three on the investigation Stretching a Penny (Appendix 2.5A), the teacher can circulate, observe, and assess: Learning Skills such as Teamwork and Thinking/Inquiry Skills by examining student's ability to pose and answer questions, and to make and verify predictions.

The two problems posed in Appendix 2.5C could be used as assessment activities. Students could work individually or in pairs and either present their solutions or submit a written solution. If class presentations are used then peers could assess the organization of data into graphs as well as the students' use of correct mathematical words. For either written submission or presentation, the teacher could assess the students' ability to create the equation of a linear relation from a context, and the effective use of technology.

Resources

Coxford, A., et al. *Contemporary Mathematics in Context*. P.O. Box 812960, Chicago, Il 60681: Everyday Learning Corp., 1997. ISBN 1-57039-475-X

MCTM (Montana Council of Teachers of Mathematics)/SIMMS. *Integrated Mathematics: A Modelling Approach Using Technology*. 401 Linfield Hall, Bozeman, MT 59717-2810: Simon & Schuster Custom Publishing.

TI. *Explorations with the CBR™*.

Appendix 2.5A: Stretching a Penny

Calculate the first differences by filling in the following table.

Number of pennies	Distance to the Plate (m)	First Differences (m)	Stretch per Penny (m)
0			
5			
10			
15			
20			

Questions:

1. Is this data linear? Justify your answer?
   Enter your data from your table to Lists in your calculator.
   Turn STAT PLOT on and graph the data. Does it appear linear?
2. From your table, what do the First Differences represent?
3. Calculate the stretch per penny for each First Difference.
4. Calculate the average stretch per penny.
5. What is the average rate of change in this relation (the steepness of the line)?
6. Using *d* for distance and *p* for number of pennies, predict the equation for this relation.
   Test your hypothesis by replacing *y* for *d* and *x* for *p* in your equation and entering this in your calculator using the Y_1= key. Does your equation produce a fitting model? If not, modify your equation.

   The calculator has a built-in feature that allows it to compute best-fitting line through a set of data. This procedure is called a linear regression. To perform a linear regression on the data you have entered in your lists, press 'STAT', 'CALC'. Select LinReg, L_1, L_2, Y_2 (VARS,YVARS, FUNCTION,Y_2). Press ENTER.
   How close is the regression equation in Y_2 to your equation in Y_1?
   Press GRAPH to display the data, your line in Y_1 and the regression line in Y_2, all on the same screen.
7. Suppose you had 100 pennies on the plate. Calculate the amount of stretch for this number.
8. a) Grab a big handful of pennies and place them gently on the plate. Trigger the CBR™ and collect the distance as you did earlier. Record this distance.
   b) Calculate the number of pennies.
   c) Count the number of pennies on your plate. How close was your prediction?
9. Suppose that the stretch was 0.5 m. How many pennies would have been on the plate? Confirm your prediction.
10. Suppose you had $12.52 worth of pennies (assuming a slinky could stretch indefinitely), how long would they stretch the slinky?
11. How would using quarters affect the steepness of the line?

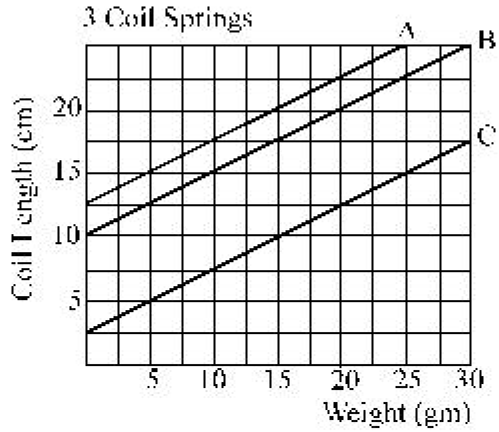
Appendix 2.5B: What's My Spring

For the following spring graphs determine the equation of each relation by identifying the slope and *y*-intercept.

1. Three Coil Springs

Identify the independent and dependent variables. Identify the slopes and *y*-intercepts. Why are the slopes the same? Why do the lines start at different points? Find the slopes then write equations for the relations. What is the physical meaning of the slopes and the *y*-intercepts?

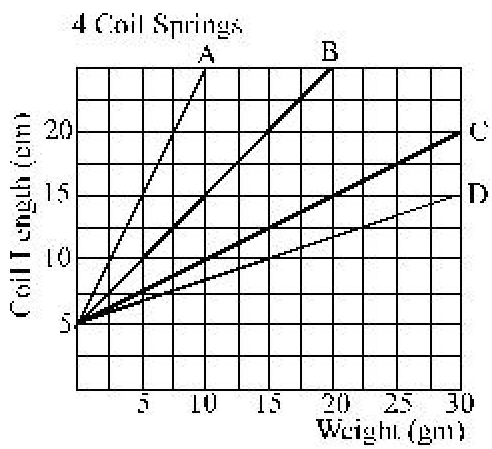
Using your equations determine the lengths for the three springs if a 5 g mass is attached to each of the springs. Check your answer using the graph. Determine the length of each spring if 30 g is attached to it.



2. Four Coil Springs

Identify the independent and dependent variables. Identify the slopes and *y*-intercepts. Why are the slopes different? Why are the *y*-intercepts the same? Find the equations for the relations. What is the physical meaning of the slopes and the *y*-intercepts?

Using your equations determine the lengths for the four springs if a 10 g mass is attached to each spring. Check your answer using the graph. Determine the length of the springs if 50 g is attached to each of them.

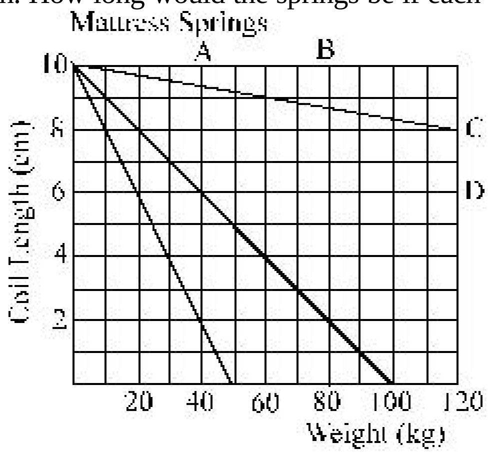


Appendix 2.5B: What's My Spring (Continued)

3. Mattress Springs

Identify the independent and dependent variables. Why is the length of the spring getting shorter? How will this be identified in the value of the slope? Identify the slope and *y*-intercept. Why are the slopes different? Why is the *y*-intercept the same? Find the equations for the relations. What is the physical meaning of the slope and *y*-intercept?

Using your equations determine the lengths if each spring has a mass of 20 kg placed on them. Check your answers using the graph. How long would the springs be if each had an 80 kg mass on it?



Appendix 2.5C: Temperature Applications

Crickets Beat the Heat

Crickets make chirping sounds by rubbing their wings together. For some crickets, the relationship between the number of chirps per minute and the air temperature is very close to being linear. Use the following table of values to answer the given questions.

Temp (°C)	18	20	22	25	27	30
Number of chirps	152	124	181	204	172	240

1. Graph this relation on a scatter plot. Let x represent the temperature in degrees Celsius and y represent the number of chirps per minute.
2. Draw a line that fits the data as closely as possible.
3. Identify the y-intercept of the line. What does this intercept suggest? Is it reasonable to extrapolate the data to this point?
4. Write an equation of the line in the form $y = mx + b$.
5. Enter your data points onto a graphing calculator or computer. Graph the equation of the line. Does the equation produce a fitting model? If not, go back and make the necessary adjustments.
6. Predict the temperature at which these crickets make 160 chirps per minute.
7. How many chirps they make if the temperature is 23°C?

A Frozen Balloon

When an inflated balloon is placed in a freezer, its volume decreases as the air inside it grows colder. When the balloon is removed from the freezer, its volume increases as it warms. The following table shows some data comparing the temperature of air in the balloon to its volume.

Temperature (°C)	Volume (mL)
5	513
10	520
15	532
20	544
25	550
30	559
35	569
40	578

1. Make a scatter plot of this data. Let y represent the volume in millilitres and x represent the temperature in degrees Celsius.
2. Draw a line that closely approximates the data.
3. Write an equation of the line in slope/intercept form.
4. Use your equation to predict the volume of the balloon at an air temperature of 100°C.
5. Extrapolate back to find the x-intercept. (You may need to adjust the values in your window to do this.) What is the physical meaning of this?

Activity 2.6: Sunshine, Whiskers, and Windmill

Time: 150 minutes

Description

Students investigate the slopes of parallel and perpendicular lines by graphing equations in the form $y = mx + b$ using a graphing calculator or computer. They also investigate lines with the same *y*-intercept, lines symmetric about the *x* and *y* axis, and vertical and horizontal lines.

Strand(s) and Expectations

Strand(s): Number Sense and Algebra, Relationships, Analytic Geometry

Specific Expectations: NA3.05; RE1.05; AG2.02, .04; AG3.02, .03, .04, .05.

Planning Notes

- This activity could be done in pairs or small groups.
- Graphing calculators or computers are needed for each pair or group of students

Prior Learning Required

Graphing lines using technology; Identifying the slope and *y*-intercept from an equation

Teaching/Learning Strategies

Teacher Facilitation: The teacher may wish to have students write all they know about equations and graphs of lines in about two minutes of individual work. Gather students' ideas and organize them on the blackboard. This full class review of everything students know about equations and graphs of lines is followed by small group investigation.

Student Activity: What's My Property?

As you work through the following investigations make notes of what you have discovered.

TI-83+ Instructions:

- Turn on grid points: FORMAT (2nd; Zoom); highlight GridOn; ENTER
- Set the WINDOW parameters as follows: Xmin = -5; Xmax = 5; Xscl = 1; Ymin = -5; Ymax = 5; Yscl = 1
- Make the appearance of the X and Y scales the same on the calculator screen. Press Zoom; press 5 (to select the Zsquare feature)
- Now, go back to the Window screen to see what is new. Why are the X max and min values different?

Activity Instructions

As you work through the following investigations make notes of what you discover

1. a) For each group (column) of equations identify what is the same and what is different.
   Using a graphing calculator or computer sketch the groups of graphs in your notebooks, graphing each group on its own set of axes. Clear the equations from each group before beginning on the next.
   b) How are the lines for each group related?

A	B	C
$y = 3x$	$y = -2x$	$y = \frac{1}{2}x$
$y = 3x + 3$	$y = -2x + 3$	$y = \frac{1}{2}x + 3$
$y = 3x - 1$	$y = -2x - 1$	$y = \frac{1}{2}x - 1$
$y = 3x - 3$	$y = -2x - 3$	$y = \frac{1}{2}x - 3$

2. For each pair of given lines identify the slopes, graph them on the same set of axes and determine the relationship between the lines. Clear the equations from each pair before beginning on the next.

A	B	C
$y = x$	$y = 4x - 2$	$y = \quad x$
$y = -x$	$y = -4x - 2$	$y = - \quad x$

3. a) Graph the following groups of lines by creating a table of values first. Note that $y = 2$ means that the value of y is 2 for every value of x. Graph each group on its own set of axes.
   b) How are the lines in column A related? Pick any 2 points on a line from column A and calculate the slope. How does the slope relate to the graph?
   c) How are the lines in column B related? Pick any 2 points on a line from column B and calculate the slope. How does the slope relate to the graph?

A	B
$y = 2$	$x = 2$
$y = 5$	$x = 5$
$y = -2$	$x = -2$
$y = -5$	$x = -5$

4. a) Sketch the graphs of the following pairs of lines on the same set of axes. How are the pairs of lines related? How could you verify this?
   b) Express the slopes for each pair of lines in fraction form. How are the numerators and denominators related? What is the product of the slopes in each pair?

A	B	C
$y = 2x + 1$	$y = \frac{1}{4}x - 2$	$y = \quad x - 4$
$y = -\frac{1}{2}x + 3$	$y = -4x + 5$	$y = -\frac{3}{2}x + 2$

5. a) For the following lines describe what the graphs look like without graphing the line. Then, if possible, confirm your conclusions by graphing using a calculator or computer.
   b) Identify pairs of lines that are related, (i.e., parallel, perpendicular, same y-intercept, reflections). For each pair of lines, explain how the equations indicate the relationship.

$y = -1$	$x = 4$	$y = \frac{3}{5}x - 1$	$y = -\frac{5}{3}x + 4$	$y = -\frac{3}{5}x - 1$

6. Summarize what you now know about the graphs of lines and their equations. Submit your conclusions to your teacher.

Teacher Facilitation: Introduce and discuss the "negative reciprocal" relationship. Help pairs as needed. A comparison to transformations could be made (shift up/down, reflection in the x or y axis). Discuss slopes of vertical lines and x-intercepts with the whole class. This would also be an excellent opportunity to reconnect slope as a rate of change and y-intercept as an initial condition.

Follow-Up Practice and Assignment:

Teacher Facilitation: Have students practice (i) Determining equations of lines given two characteristics about the line (slope and y-intercept; y-intercept and parallel or perpendicular to a line, vertical or horizontal line and a point or intercept, given a graph containing the preceding information).
(ii) Graphing lines given equations in the form $y = mx + b$;
Have pairs of students sketch the graphs of linear relations, one student sketching by hand, while the partner uses technology. Compare results, then exchange roles.
Have students work on questions like the following to identify the role of slope and y-intercept from real life situations.

Student Activity:

1. Each equation given below models a stretching or compression experiment with a spring where L is the spring length in centimetres and m is the mass acting on the spring in grams.
   Identify the following:
   - the initial length of spring
   - the rate of change of length
   - whether the experiment was designed to measure spring stretch or spring compression

   (i) $L = 5m + 2$
   (ii) $L = 3.6m + 4$
   (iii) $L = -2.5m + 1$
   (iv) $L = -0.3m + 6$
2. The student athletic council at Valley High School operates a bottled water machine near the gymnasium. The council is paid $50 per month plus $4 per case sold.
   (i) Find an equation showing the monthly income as a function of the number of cases sold. Use your equation to calculate the monthly income if 15 cases are sold. Find the number of cases that need to be sold for an income of $138.
   (ii) What would the equation be if the council was paid $4 per case plus $60 per month? Is the new situation better or worse for the council? Explain.
   (iii) What would the equation be if the council was paid $50 per month plus $5 per case? Is the new situation better or worse than the original for the council? Explain.

Assessment/Evaluation Techniques

Student submissions of the investigation could be assessed for:

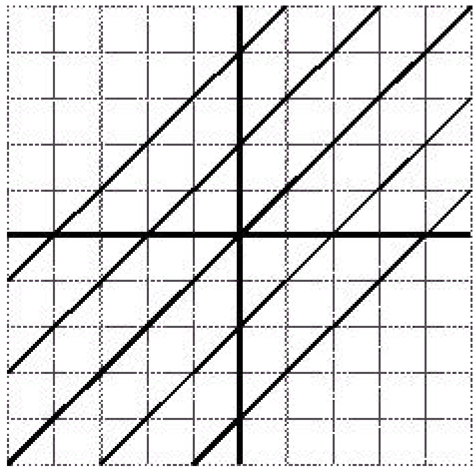
- communication and justification of conclusions about relationships between graphs
- recognition of slope and y-intercept in context
- identification of patterns, properties, and relationships of slopes
- ability to determine the equation of a line

The following assignment Line Designs also aids in the assessment of the student's ability to determine equations of lines. The student's engagement in designing their own creation could serve as an indicator of initiative.

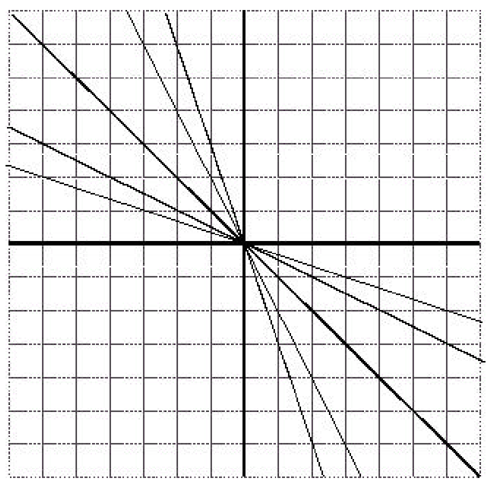
Line Designs

RAYS OF SUNSHINE

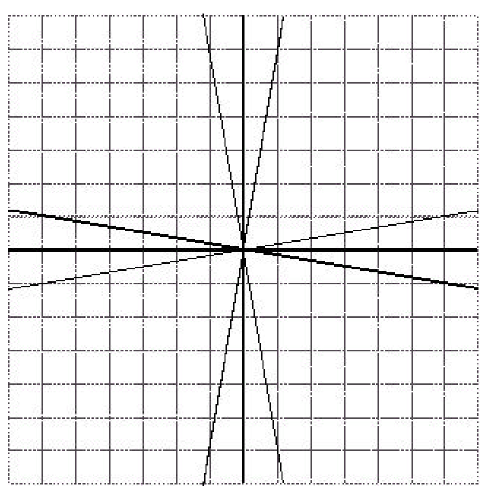
a) Write the equations of the lines that create the following pictures on a graphing calculator or computer screen and hand in your solutions.



Rays of Sunshine



Kitty's Whiskers



Windmill

b) Design a creation of your own. Include your picture and equations. Give your picture an appropriate title.

Activity 2.7: Break the Bank!

Time: 225 minutes

Description

Students investigate linear equations in y = $mx + b$ form and $Ax + By + C = 0$ form to determine that both forms of the linear equation graph the same line. This activity provides a context for revisiting the solution and rearrangement of linear equations. Students rearrange linear equations of various forms, (e.g., $y = mx + b$, $Ax + By + C = 0$, $x = a$, and $y = b$). As a follow-up, students determine the x- and y-intercepts of a line given its equation in $Ax + By + C = 0$ or $Ax + By = D$ form and graph lines given their equations in the various forms listed above.

Strand(s) and Expectations

Strand(s): Number Sense and Algebra, Analytic Geometry

Specific Expectations: NA1.01, .05; NA3.01, .03, .04; NA4.01; AG3.01, .02, .03.

Planning Notes

- Students use graphing calculators for this activity.
- Overhead transparencies with the same grid markings and intervals must be prepared ahead of time. Water-soluble overhead markers are also needed.
- Prepare a variety of sets of equations (two or three in each set) that represent the same linear graph: e.g., $x - y = 4$, $x - y - 4 = 0$, and $y = x - 4$
  or $2x + y = 4$, $y = -2x + 4$, and $-2x - y + 4 = 0$
  Put each equation on a strip of paper for distribution to pairs later in the lesson.
- Teachers should explain to their students that $Ax + By + C = 0$ and $Ax + By = D$ are both common forms for a linear equation. Both forms should be included in examples discussed in class and assigned to students.

Prior Learning Required

Students should be competent in graphing equations in "$y =$" form using graphing technology and in creating and using a table of values to graph equations not in that form. Students are also familiar with rearranging and solving equations from Activity 2.4.

Teaching/Learning Strategies

Teacher Facilitation: Begin this activity by building on the students' intuitive sense of relationships between numbers. For instance, ask students to list, as ordered pairs, five sets of numbers that have a sum of 15. (Or use a scenario such as: there are 15 coins in a piggy bank, made up of nickels and dimes.) Ask a few students to name their ordered pairs and then in a full class discussion ask how to write the equation if one number is x and the other is y. You should receive the response $x + y = 15$ but you may also receive $y = 15 - x$. If not, then ask students what the second number of an ordered pair would be if the first number was ten. Then ask how they calculated the number. Hopefully someone says something like: "You just subtract the number from 15." Talk to students about how to write this as an equation ($y = 15 - x$). Discuss with students whether the two equations are the same. These could be checked in a variety of ways: compare hand-drawn graphs; enter $y = 15$ - x into the graphing calculator, look at the table and compare it to the original table of values; re-arrange one equation to a form similar to the second. Similar scenarios could be pursued (e.g., how many of each of two kinds of donuts make up a dozen; how many boys and girls there are in a class of 30; the length and width of a rectangle if its perimeter is 24; the difference in age between John and his brother is five years;)

Using the prepared sets of equations, give one equation to each pair of students. Include equations like $y - 2 = 0$ and $y = 2$. Note that some of the equations can be done easily with a graphing calculator, while others cannot.

Student Activity:

Each pair of students graph their equation on the pre-gridded acetate using a graphing calculator or a table of values as necessary. Using the overhead, students from one pair display their graph and ask if another group has a graph that matches their graph. Place the match the graph on top to confirm. When the matching graphs are found, the presenting pair write the equations on the board.

Teacher Facilitation: Discuss patterns and similarities in each set of equations before beginning the next. Discuss which form of the equation is easiest to graph. When students choose $y = mx + b$ as the easier form for graphing, use this as a lead in to practice how to re-arrange equations. Build on their intuitive sense from the first example and from their knowledge of solving equations from Activity 2.4. Include examples with fractional coefficients as in $x + 2y = 6$ as well as examples set in context such as; converting Fahrenheit temperatures to Celsius; relating the speed of sound to temperature, $v = 330 + 0.6T$, where v is speed in m/s, 330 is the speed of sound in m/s at 0EC, and T is temperature in degrees Celsius; the perimeter of a rectangle $2(l + w) = P$, where l is length in metres, w is width in metres, and P is perimeter in metres.

Homework/Extension

Assign textbook practice on re-arranging linear equations into the forms $Ax + By + C = 0$, $y = mx + b$, $x - a = 0$, $x = a$, $y - a = 0$ and $y = a$. The focus should be on re-arranging $Ax + By + C = 0$ into $y = mx + b$ with limited practice on converting the other way.

Find slope and y-intercept of equations in the form $Ax + By + C = 0$ by writing the equation in the form $y = mx + b$.

Follow-up Algebra Lessons:

- Determine the x- and y-intercepts of a linear equation in the form $Ax + By + C = 0$ or $Ax + By = D$, and use them to graph the line.
- Graph a line given its equation in $Ax + By + C = 0$, $y = mx + b$, $x = a$ or $y = b$ form.

Assessment/Evaluation Techniques

- A rubric for observation could be used as the students complete the graphing activity, to assess students' ability to work in small and large group situations, solve problems, graph lines using a table of values, recognize patterns, and make conclusions.
- Communication skills could be assessed through journal writing, as students explain how and why they used graphing calculators or tables to graph lines, which method they prefer, and why, and when it is appropriate/necessary to graph using a table or calculator.
- Paper and pencil tasks or a short quiz could assess student's ability to re-arrange linear equations, solve equations in one variable and graph a line using x- and y-intercepts.

Activity 2.8: Fireworks and Twinkle, Twinkle

Time: 75 minutes

Description

Students find the equation of a line using two points on the line. They recognize that any two points on a line lead to the same equation. Using this new skill they then determine the equations for distance-time data collected using a CBR™ or CBL. This activity also offers an extension leading to piecewise functions.

Strand(s) and Expectations

Strands: Analytic Geometry, Number Sense and Algebra, Relations

Specific Expectations: AG2.01, .04; AG3.03, .04, .06, .07; NA1.03; NA3.04; RE1.03, .04, .05.

Planning Notes

- Graphing calculators are required for this activity.
- Photocopies of Canada Day Fireworks and Twinkle, Twinkle for each pair of students.
- Each group needs access to at least one graphing calculator and a CBR™.
- If using a CBL, then *Match It*, *Graph It* program from the *Real World Math* Book from TI needs to be down loaded into their calculators.

Prior Learning Required

Students need to know how to calculate slope, enter lists and graph using "y =" in a graphing calculator, isolate a variable in an equation, or substitute into a formula, then solve.

Teaching/Learning Strategies

Teacher Facilitation: The activity begins with a full-class discussion interspersed with individual or pair student work on the following three examples.

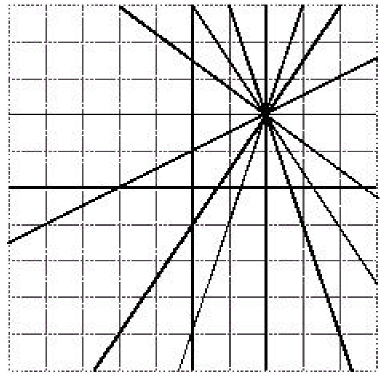
In Example 1 Canada Day Fireworks, students can find the equation of the lines by using the skills developed in Activity 2.6. Some are simply calculating slope and reading the y-intercept from the graph. This gives them the $y = mx + b$ form. Tell students that the interval on each axis is one unit.

In Example 2, Twinkle, Twinkle, it is difficult to read the y-intercept and therefore another method for finding the equation of the line must be used. At this point, the teacher should interrupt the work on Example 2 and direct the students to work through Example 3, which leads students through the development of finding the equation of a line, given two points on it.

In Example 3, each pair should be assigned a specific point as their starting point. After students complete step 3 of Example 3, the teacher should interrupt the class and have students put their groups' solutions on the board or chart paper and compare the slopes. Since all groups get the same result the class can now generalize this by letting P(x,y) be any point on the line. Students should then be directed back to the example and complete the rest of the steps.

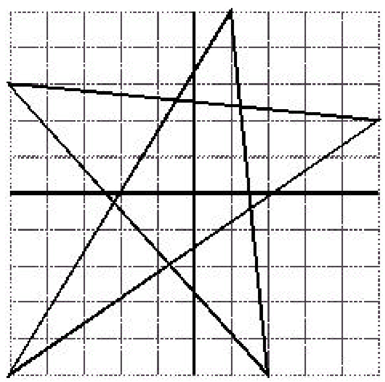
After students have obtained the equation of the line using their point, groups should put their solutions on the board or chart paper and compare the initial equations and the final equations. Students may need help simplifying their equations. The purpose of the activity is for students to see that any two points on a line will lead to the same equation. After completing Example 3, teachers and students should summarize finding the equation of a line given two points. Students should then revisit Example 2 and find the equations of all of the lines in the Twinkle, Twinkle grid.

Student Activity:



Example 1 (Canada Day Fireworks)

The picture shows the graphs of several different lines. Find the equations of the lines and graph them on a graphing calculator, if possible. Make sure that your window matches the diagram. Graph them on your calculator.



Example 2 (Twinkle, Twinkle)

Find the equation of each line. Notice that although it is easy to locate two points on each line, it is difficult to locate the *y*-intercept. Do you have some ideas as to how to find the equation?

Example 3 (Publishing Yearbooks)

The Mustang Publishing Company produces yearbooks. Their charge is based on the number of books ordered. The following chart is an example of the costs.

Number of books	200	400	600	800	1 000	1 200
Cost	3 500	6 500	9 500	12 500	15 500	18 500

1. Enter this data into lists on a graphing calculator and determine if the relation is linear.
2. Take the starting point assigned to you by the teacher and call it (x_1, y_1). Name the point to the right of your point (x_2, y_2). Find the slope of the line using: $\frac{y_2 - y_1}{x_2 - x_1} = m$.
3. Change the order of the subtraction i.e., $\frac{y_1 - y_2}{x_1 - x_2} = m$ and determine if it will give you a different result.
4. Repeat step 2 from above by replacing (x_2, y_2) with (x, y). In other words, substitute your slope value for m and your point for (x_2, y_2) in $\frac{y - y_1}{x - x_1} = m$.
5. If you wanted to graph this you would need to put it in the form of "y =". Re-arrange the equation to isolate y.
6. Graph the equation on your graphing calculator. Does it match your original relation created from lists?

Follow-up Activities:

Student Activity:

Students collect real-life distance-time data using CBL or CBR™. They determine the equation of the line describing their motion using the previous method, (i.e., they should find the slope of their line and let (x_1,y_1) be any point on the line). If (x, y) is any other point on the line, then the equation of the line would be: $(y - y_1)/(x - x_1) = m$.

Their equation can now be entered into the calculator and graphed. They need to identify the domain for each section. This verifies the correctness of their model. Students give the meaning of the slope and y-intercept if appropriate. Using their equation they are asked to find the distance walked for several different time periods. This could be checked on their calculators using the trace key. They should also answer questions that require them to find the time taken to travel different distances using their equation. Once again their answers can be confirmed using the trace key.

Extension:

Teacher Facilitation: For some students ready for extensions, they can investigate how to graph piecewise functions through the following example.

Example: Speedy Copy charges the following rates for photocopying:

- 3 cents each copy up to and including 500 copies
- 2 cents for each additional copy beyond 500.

1. Draw a table of values and graph the relationship between the number of copies and the cost.
2. Notice that the graph is broken into two sections. Calculate the slope of each section of your graph.
3. Calculate the equation for each section of the graph.
4. Notice that we could obtain the equation by using the re-arrangement of $\frac{y - y_1}{x - x_1} = m$

into $y = y_1 + m(x - x_1)$ form where y_1 is the initial value of y and x_1 is the initial value of x for each section. This approach is useful when defining the equations of segments of piecewise linear functions which do not pass through the y-axis (e.g., for the Speedy Copy charges in the example above).

$$y = \begin{cases} 3x,\ 0 \le x \le 500 \\ 1500 + 2(x - 500),\ x < 500 \end{cases}$$

Student Activity:

Describe a scenario that would produce a piecewise graph similar to the photocopy charge graph. Find the equation for each piece of the graph.

Homework/Practice Suggestions:

1. Create a linear design on a grid and determine equations for the design. Ask another student to graph the equations to see if it produces the suggested linear design.
2. Using the data from Activity 2.5: Crickets Chirp to the Beat of the Temperature, determine the equation of the line by choosing two data points and compare it to your earlier model.
3. Investigate a situation of your own choice that would be modelled by a linear equation. Submit a report that includes the following:
   a) Describe your data. How was it collected? (Using probes; taking measurements (e.g., distance, height, temperature, etc.) found in magazines, reference books, the Internet, newspapers.)
   b) Sketch a graph of your data. Explain why the data is linear. Draw a line of best fit. Find its equation.
   c) Interpret the real-life meaning of the slope, y-intercept (and x-intercept, if there is one).
   d) Use your equation to predict the value of a point between two given values and beyond the given value.

e) Are there any restrictions or limitations to your data values? If so, explain why?
f) Suggest a possible use for your equation.

Assessment/Evaluation Techniques

Teachers could assess students' communication in mathematics and problem-solving skills through observation while students are working on pair activities. Students could submit their solutions to some of the above activities for assessment of the relevant expectations such as:

- determining the equation of a line given two points
- graphing lines by hand or using a graphing calculator
- communicating solutions to multi-step problems
- describing the meaning of slope and y-intercept in context
- describing a situation that can be modelled by a liner relation.

References

Real-World Math with the CBL System

Texas Instruments. *Explorations using the CBR™*.

Data Analysis and Statistics. NCTM Addenda Series, p. 36.

Zap-A-Graph Tutorials

Green Globs

Math Trek

Activity 2.9: All in the Family

Time: 75 minutes

Description

Students complete a worksheet activity investigating the characteristics that distinguish a linear relation from a non-linear relation. They use calculators or graphing software to obtain the graphs of a variety of linear and non-linear relations from their equations; classify the relations according to the shape of their graphs, and determine the characteristics of the linear equations that differ from the non-linear equations.

Strand(s) and Expectations

Strand(s): Analytic Geometry, Number Sense and Algebra, Relationships
Specific Expectations: AG1.01, .02, .03; AG3.03; NA2.01, .02; RE2.06; RE3.03.

Planning Notes

- Reserve a class set of graphing calculators or time in the computer lab.
- Prepare individual copies of the worksheet found in the Appendix. Provide each student with a sheet of graph paper to fold into four sections for graphing the four types of equations.
- Help students to suitably set the domain and range of the calculator.
- Prompt students in questions #4 and #5 to rewrite the equations in the form $y = mx + b$ to facilitate the use of a graphing calculator.
- Prompt students in question #4b to use a table (or prior knowledge) to graph equations of the form $Ax + C = 0$ as these lines cannot be graphed on a graphing calculator.

Prior Learning Required

Students already have competence graphing linear and non-linear relations from data gathering and from descriptions of realistic situations. Finite differences from a table of values have been examined for both linear and non-linear relations in Unit 1 (Phase 1).

Teaching/Learning Strategies

Teacher Facilitation: As a brief introduction and review, ask students to draw, in the air, one of the shapes of the graph of a relationship in Unit 1 or 2. The teacher could name activities like Activities 1.5 A Cagey Problem (parabolic), 1.7 Fold It! (exponential), and scenarios from Activities 2.1 or 2.4 (linear) to remind students of the different types of relationships they have encountered. Draw these shapes on the blackboard and label them. Tell students that graphs that share a property are referred to as a family of graphs.

Student Activity:

Using the student worksheet All in the Family found in Appendix 2.9, students use graphing calculators or software to classify the graphs and make conclusions about the difference between the linear and non-linear equations.

Teacher Facilitation: Ensure that the students have determined the characteristics of the equation of a line and can recognize the linear equations from a list of equations. Using the calculator, have students examine the table of values for both a curve and a line, then calculate first differences for a linear relation, and further differences for other types of relationships (2^{nd} differences are constant for parabolas, 3^{rd} differences constant for cubics, no differences constant for exponentials). Give students a table of values and ask them to calculate finite difference to determine if the relation is linear or non-linear.

Homework/Extension:

Give textbook practice in recognition and classification of linear and non-linear equations. Many students are able to recognize the equations as parabolas, cubics, and exponentials, although that level of specificity is not required.

Assessment/Evaluation Techniques

- Teachers could observe student performance during the investigation and pay attention to students' perseverance to the task as an indicator of initiative.
- Paper and pencil tasks or a short quiz could assess students' ability to recognize linear equations and classify equations and graphs.
- Communication and the application of new knowledge about classifying relations could be assessed through journal entries describing the characteristics of a linear equation that make it differ from non-linear equations. Students could also verbalize or write about the characteristics of a non-linear graph determined by second degree, third degree and exponential equations.

Appendix 2.9: All in the Family Worksheet

1. The equations below represent 4 different shapes/families of graphs. Use a computer or a graphing calculator to draw the graph of each relation. Fold a piece of graph paper into 4 sections. Place graphs of the same family on the same grid. Write the equation on top or beside each graph as you draw it.

$y = x^2$	$y = x^3 + 4$	$y = x + 4$	$y = -2x^2$	$y = (x - 3)^3$
$y = -3x + 1$	$y = 2^x$	$y = \quad x - 2$	$y = 3^x$	$y = (x + 3)^2$
$y = \quad x$	$y = (\frac{1}{2})^x$	$y = (x - 12)^3 - 13$	$y = -x - 3$	$y = 2x^2 - 1$
$y = x^3 + 2$	$y = (x - 2)^2 - 3$	$y = -2 + x$	$y = (2x)^3$	$y = \frac{1}{2}x - 3$

2. Examine the 4 families of graphs you sketched. Write the equations of any 6 graphs that are linear. Write the equations of any 6 graphs that are not linear. Compare the equations of the lines with the equations of the others. How do the equations of the lines differ from the equations of the others?
3. Complete this statement about how you would recognize an equation that graphs a line:
   "An equation graphs a line if..."
4. a) Graph these equations that contain a *y* variable only:

$y = 5$	$y + 2 = 0$	$2y = 1$	$y = 0$	$3y + 6 = 0$

   Are the graphs linear? What special characteristic do each of the lines possess? Make a statement about the appearance of a graph if its equation has a *y* variable only.

   b) Repeat the questions in #4a above for these equations that contain an *x* variable only:

$x = -1$	$x + 3 = 0$	$3x - 1 = 0$	$x = 0$	$2x + 1 = 5$

5. Put checkmarks on the equations that graph lines. Use a graphing calculator to graph the equations that you checked to verify that they are linear. Predict the shapes of the other graphs and check using a graphing calculator.

❑ $y - x = 0$	❑ $y = 1 - x^2$	❑ $y - 2x^2 = 3$	❑ $y + x + 1 = 0$	❑ $y - x^3 = 5$
❑ $y = (x - 1)^2$	❑ $y = 5^x$	❑ $0 = 2x + y + 3$	❑ $y = -3x^3$	❑ $y = x - 2$

6. Make up 5 equations of your own that graph lines. Graph your 5 equations to ensure that they are linear equations. Graph a variety of lines that have different directions.
7. Examine the exponents of the different types of equations that graph non-linear relations. Is there a link between the value of the exponent and the shape of the graph? Make a prediction about the link and test your prediction by making up and then graphing several examples. Did you verify or refute your prediction? If you refuted the prediction, make another prediction and test it.
8. Make up some equations that graph curves that do not belong to any of the types examined today. Sketch their graphs.

Activity 2.10: Planning a Trip - a Set of Summative Assessment Activities

Time: 225 minutes

Description

In this activity, students use given data and information to create mathematical models, analyse choices, and make decisions. They use the skills developed in Unit 2 to answer a series of questions that connect to the theme of planning a trip. The organization of these summative assessment tasks is similar to those in Unit 4. These questions help prepare students for the type of questioning that can be expected at the end of the Grade 9 course. Time is also set aside for students to learn, practise, and take pencil and paper tests of algebraic skills.

Strand(s) and Expectations

Strand(s): Analytic Geometry, Number Sense and Algebra, Relationships
Specific expectations: AG1.03; AG2.03, .04; AG3.01, .02, .03, .04, .05; .05, .06; .04; NA4.01, .03; RE2.01; RE3.03, .04.

Planning Notes

- The Planning a Trip activities could be used over a week, where half of the class time is spent reviewing key skills and the other half is spent applying knowledge to these activities. Or, this set of summative assessment tasks could be concentrated in the last three days of the Unit. It is estimated that Planning a Trip Phases 1 and 2 requires about 30 minutes each and Phases 3 and 4 about 15 minutes each. A traditional pencil and paper test could follow these performance assessment and review sessions.
- It is suggested that students gather their written analyses in a portfolio. All students should become familiar with the quality of work that is judged to be Level 3 and 4. This could be achieved through discussion of the characteristics of student performance and written submissions that would be Level 3 or 4, before the activities are undertaken. Or, there could be some full class presentations and discussion of students' findings after each activity is assessed. Teachers should make specific suggestions to each student as to how their work in various categories could be improved.

Prior Knowledge Required

The students have completed the work of Unit 2 of the Profile.

Teaching/Learning Strategies

Student Activity: Planning a Trip

Mr. And Mrs. Lee and their three children, Todd, Chris, and Vikki are planning to take an extended holiday (perhaps four - six weeks) a year from now. They want to drive their van from <your town here> to Vancouver, visiting many points of interest along the way. The Lees know that they have to budget and save all year to be able to afford the type of trip they want. Over the next few classes, you will be given details that the Lees want to analyze to develop their budget for the trip. Keep your analyses together in a "trip portfolio".

Planning a Trip Phase 1: Kennel Costs

The Lees need to board their pet cat, Bailey, while they are on their holiday. It is Chris's job to research the fee structures of local kennels. He clips the following ads from his local newspaper.

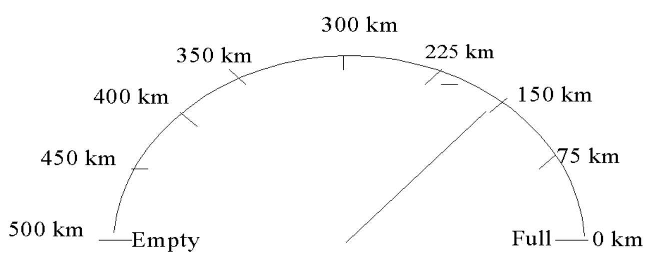
1. Create a graphical model for each of the kennels that is appropriate for Bailey. Show all graphs on the same grid and label them clearly.
2. Form the equations for the fee structures at the kennels you think the Lees should consider for Bailey. Define the variables you use.
3. Use your equations to contrast the kennel charges if the Lees think that they will be leaving Bailey for either 4 or 5 weeks. Label your computations clearly.
4. Where do you think Chris should recommend the family board their cat? Explain your reasoning.
5. Later, Chris sees an ad for a new kennel in their neighbourhood. It charges $5 for an initial flea check, plus $10/day. Chris rejects this kennel without creating a mathematical model or computing costs. Explain Chris' reasoning for rejecting this kennel so quickly. Use the vocabulary of this Unit as you make your points.

Teacher Facilitation:

- Students work individually on this activity.
- The teacher circulates around the classroom as students engage in this activity, giving prompts with a coloured pen if a student is having trouble engaging the problem, or making a misinterpretation. Teacher prompts are taken into consideration when the student's work is graded.
- As students work on this activity, ensure that they do not spend much time on the Bow Wow Weekend ad, since that kennel accepts only dogs.

Planning a Trip, Phase 2: Gas Up!

On a 3-day visit to their grandparents in New York, the Lees drive their van, and gather data to analyse the rate of fuel consumption of the van and the accuracy of its fuel gauge. They fill the tank up with gas and set the trip odometer to zero as they set out. It is Todd's job to record the trip odometer readings as the gas gauge reaches each of the eighth markings on the gas gauge. Todd records his readings below.



1. Create a table of values using the odometer reading as the independent variable.
2. Plot the data on a carefully labeled and scaled grid.
3. What does this data suggest about the rate of fuel consumption of the van?
4. Based on this data, what comment could you make about the accuracy of the fuel gauge?

5. What would you expect the gas gauge to read at the instant the Lee family had driven the following distances from their last gas up? Show your work in a format that is easy to follow.
   a) 100 km b) 415 km
6. How far from their last gas up would you expect the Lee family to be when their gas gauge reads 1/3 full? Explain your reasoning.
7. The route the Lee family plans to take for their trip is 7500 km on the way out to Vancouver and 7300 km on the way back. How many tanks of gas might they expect to need for their trip? Explain your reasoning.
8. The low gas warning light comes on when the van's gas gauge reads 1/16 full. The Lees always gas up just before or just after the warning light comes on. How many fill ups will the Lees need for their holiday?
9. Pose other questions that would fit the context of a trip in your family vehicle.

Teacher Facilitation:

- Ensure that students know that an odometer measures distance traveled and that the needle on a gas gauge rotates counterclockwise as the gas tank goes from full to empty.
- Students work individually on this activity.
- The teacher should circulate around the classroom giving prompts with a coloured pen if a student is having trouble engaging the problem or is misinterpreting a question.

Planning a Trip Phase 3: Lawn Care

The Lees usually tend their lawn and gardens themselves. However, during their holiday, they plan to engage either the Green Grass Company or the Careful Cutters Company because they notice that both companies work in their neighbourhood.

When they called Green Grass for a price quote they found out that the costs are:

$90 start-up, plus $20/hr for lawn care

They tried phoning Careful Cutters several times but were only ever able to get an answering machine and no one called them back. They were persistent because two of their neighbours claimed that Careful Cutters gave the best deal. Todd was asked to investigate. He went to one neighbour who used Careful Cutters and found out that after Careful Cutters had worked for them for 22 hours, the cost was $575. Another neighbour claimed that after their first 11 hours they were charged $300.

1. Graph the relationship between the number of hours and cost for the two companies on the same grid.
2. Form an equation for each lawn care company, using h as the number of weeks for the contract to run and c as the cost of the service. What assumptions are you making about the Careful Cutters price structure (relationship)?
3. Write an advertisement for the Careful Cutters Company and state the start-up cost and the cost per hour in the advertisement.
4. The Lees estimate that their lawn requires approximately 1.5 hours of care per week. If they are gone for 6 weeks, how much would each lawn care service cost? 4 weeks? 1 week?
5. Which company is less expensive? Explain your answer.

Teacher Facilitation:

As students work on this activity, remind them to use h as the independent variable, and to identify an initial point and rate for each company. Indicate prompts using your colourful pen.

Planning a Trip Phase 4:

1. Vikki knows that her father prefers to see graphs of data when he wants to make comparisons. He gives her a file folder of ads that he has gathered and asks her to do quick sketches for him. Describe the graphs that Vikki should draw for each ad by referring to as many of the following descriptors as apply:
   a) linear b) non-linear c) increasing d) decreasing
   e) a set of parallel lines f) a set of lines that all pass through the same point
   i) **Security Guard - a Home Protection Service**
   Plan A: $5/day; Plan B: $50 labels for windows, plus $5/day; Plan C: $100 permanent registration fee and labels, plus $5/day
   Descriptors that apply: ________________________
   ii) **Pool Cleaning Companies**
   Company A: $50 for the first 2 weeks, plus $25/week thereafter
   Company B: $50 for the first 2 weeks, plus $30/week thereafter
   Company C: $50 for the first 2 weeks, plus $20/week thereafter
   Descriptors that apply: ________________________
   iii) **Reliable Power Battery Company**
   Our 9-volt batteries maintain their voltage extremely well for the first 3 weeks of constant use, then quickly drop off, resulting in a dead battery after 4 weeks.
   Descriptors that apply: ________________________
2. Describe the type of relationship that would model:
   i) the distance from Vancouver vs the number of hours away from <your home town>(as the Lees drive out to Vancouver).
   ____
   ii) the amount of money spent by the Lee family as the number of days of their holiday increases
   ____

Assessment/Evaluation Techniques

Teachers could assess the students' trip portfolios and could concentrate on any of the following areas:

Phase 1

Knowledge: correct graphs, correct equations, recognizes slope and *y*-intercept and uses them to form equations

Communication: clearly explains recommendations

Phase 2

Thinking/Inquiry - reasoning, justifying answers

Knowledge - unit rates, meaning of slope

Phase 3

Knowledge: forming equations given two points

Application: creating equations from given scenario and applying equation to answer questions.

Phase 4

Communication: used descriptors/terminology appropriately.

Accommodations

- Break activities into smaller parts, increase timelines, help student to organize each task

Appendix 2.10: Suggestions for Questions for Summative Assessment

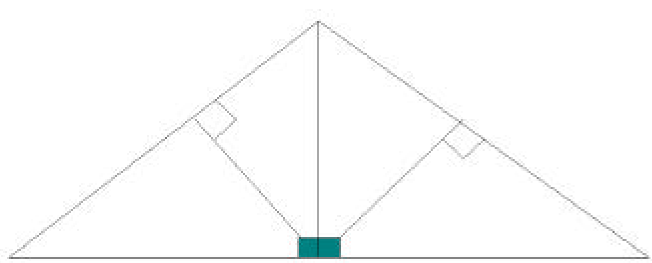
The following questions are suggestions for inclusion in a summative assessment. They require that students extend and apply their knowledge and offer opportunities for Level 4 performance. Teachers may want to choose one question to be included in a test or may want to choose one question for students to work on in groups as a performance task.

1. Relations are often represented by equations, tables of values, and graphs of lines or curves. It is possible to use equations and graphing to construct geometric patterns with lines or curves as well. For each set of lines given, determine if the lines form the sides of a square, rectangle, rhombus or parallelogram. Before you graph the lines, make a guess as to what shape the lines form. Record this guess as well as your reasons for guessing that figure. Then draw the graphs and make a statement as to whether your guess was accurate. If you guessed incorrectly, suggest what you overlooked in your first analysis.

   Case a) $y = \frac{x}{3} - 2$, $y = -3x + 18$, $y = \quad x + 18$, $y = -3x - 12$

   Case b) $2x - 3y + 15 = 0$, $x + 3y - 24 = 0$, $4x - 6y - 6 = 0$, $x + 3y + 3 = 0$

- The diagram below is of a roof truss for a cottage. Apply your knowledge of slope and the equations of lines to reproduce the identical pattern on a graphing calculator and/or graph paper. What are the equations of the lines that are involved?



Unit 3: Measurement and Dynamic Geometry

Time: 30 hours

Unit Description

Formulas for two- and three-dimensional shapes are used to solve problems involving composite plane figures. Students investigate relationships between area and perimeter, and surface area and volume, to determine optimal values within a given context. A project and a summative assessment activity provide rich contexts for applications of measurement skills.
Geometric relationships involving two-dimensional figures are explored in a variety of ways. Conjectures are made and tested using dynamic geometry software. Teachers bring closure to investigations and guide students to form generalizations of their findings. Students' knowledge is affirmed through the application of geometric relationships in new situations.

Strand(s) and Expectations

Number Sense and Algebra Specific Expectations: NA 1.01, 1.02, 1.03, 1.04, 1.05, 1.06; NA 2.02; NA3.04, .05, .06; NA4.01, .02, .03.
Relationships: RE1.03, .04, .05, .06; 2.04, 05; 3.04.
Measurement and Geometry Specific Expectations: MG1.01, .02, .03, .04; MG2.01, .02, .03, .04, .05; MG3.01, .02, .03, .04, .05.

Activity Titles

What follows are suggested packages of activities for teaching Unit 3, with timing for each activity and timing for skill development. The measurement activities could be done before or after the geometric relationships involving two-dimensional figures, allowing different Grade 9 classes to use computer facilities at different times. This profile develops only the activities that depart from traditional pencil and paper skill development. These activities are designed to develop students' ability to visualize and analyze geometric shapes and relationships, using numeric and algebraic skills as well as skills in the appropriate use of technology. Skills developed through the Activities are indicated in [brackets].
*Up to 525 of the 1800 minutes for this Unit may be used as needed for consolidating skills and further evaluation.

Activity 1	**Fence Me In! - Introduction to *The Geometer's Sketchpad*™** [maximizing area for a given perimeter, perimeter and area relationships, estimation skills]	150 minutes
Activity 2	**The Size Is Right! Three-dimensional Optimization** [comparing volumes and surface areas for square-based prisms and cylinders; optimal surface area, developing the formulas for the volume and surface area of cones, pyramids and spheres; numeracy skills]	150 minutes
Activity 3	**Why Do Elephants Have Big Ears?** [surface area: volume ratios for cubes, optimizing surface area of rectangular prisms with fixed volume]	150 minutes
Activity 4	**Parks and Recreation** [a summative assessment activity that uses the skills and knowledge from Activities 1 to 3]	150 minutes

*Time for: practicing the use of geometric formulas, re-arrangement of formulas, presentation of projects. 300

Activity 5	**Exploring with *The Geometer's Sketchpad*™** [review of Angle Relationships]	150 minutes
Activity 6	**If…then…** [Investigating Geometric Properties Using *The Geometer's Sketchpad*™]	300 minutes
*Time for: further investigations, assessments.		225 minutes
Activity 7	**Script It!** [creating *The Geometer's Sketchpad*™ Scripts; suggestions for summative assessment activities]	225 minutes

Prior Knowledge Required

- Measurement and Geometry and Spatial Sense skills from elementary school.
- Unit 1: determine the relationship between two variables by collecting and analysing data, solve multi-step problems requiring numerical answers

Unit Planning Notes

- Check the availability of a computer lab before starting Unit 3. Most of approximately eight 75-minute classes would be best spent having students work in pairs on a computer. It is possible to re-arrange the order of some of the activities within this Unit. If computer lab time is not available, it would be possible to capture the power of dynamic geometry software using: one computer attached to a projection panel or TV, or one TI-92 calculator attached to a Viewscreen.
- Each pair of students working on *Sketchpad* can work at their own pace, but the teacher needs to ensure that all pairs get the help they need to complete the required investigations. Pairs who complete the investigation more quickly could be challenged with extensions, or to pose "What if...." questions; their investigations could be reported to the class. Using technology, each pair of students is responsible for covering all of the special cases and a large number of examples.
- The Lawn Sprinkler Activity in the Applied Profile would be an excellent pre-activity on 2-D shapes and measurement.
- If a class cannot use a computer lab, it is possible to carry out the investigations in Activities 5 and 6 using careful constructions and measurements, by hand. If this is the case, the teacher must ensure that students understand that many more examples are needed than they, individually, have time to create, to test each hypothesis adequately. The pooling of results of the entire class will have to be orchestrated. It is important that each pair of students understands what extreme cases need to be investigated for each hypothesis. Textbooks suggest ways to teach this material if your department does not yet have computer access for your class.
- Before starting Activity 3, the class gather items such as tin cans, or other cylindrical shapes, in various sizes, plus a large quantity of plastic pellets, rice, or other small hard objects
- Activity 5 is intended to introduce the use of *The Geometer's Sketchpad*™ and review relationships from Grade 8. Subsequent activities introduce new relationships using *Sketchpad* as an investigative tool.
- Teachers should ensure that they know what mathematics needs to be drawn out of each of the activities and plan enough time at the end of each class to help bring closure to the investigations that students have performed.

Teaching/Learning Strategies

Teachers focus the students' previous graphing and identifying-relationship experience to connect the measurement activities of this unit back to Units 1 and 2. This brings the course around full circle and prepares the students for Grade 10.

Pairing works well as students learn to use the dynamic geometry software. Make sure that each student spends enough time using the keyboard to become comfortable with the software.

Move from pair to pair as the students carry out their investigations on the computer. By listening to what the students are discussing, it is possible to ensure that they are forming good hypotheses, and drawing appropriate conclusions. By asking probing questions, it is possible to be certain that each pair of students is getting as much out of each investigation as they should.

Test the computer constructions for each pair of students by asking students to create and save scripts or by performing a drag test (If the diagram has been properly constructed, the structure stays connected when one of the elements is moved by clicking on the mouse and dragging it.) on their screen.

There are some activities that are different in the Applied and Academic versions to reflect different expectations.

Assessment/Evaluation Techniques

As in Units 1 and 2, a variety of assessment tools and strategies is recommended. Performance assessments may be used to effectively assess Thinking/Inquiry/Problem Solving, Communication, and Application Categories of the Achievement Chart when students do open-ended tasks. Learning Skills can be assessed using teacher- and peer-observation, and self-reflection. Rubrics and rating scales are useful when a wide range of performance is expected and when many complex criteria are to be judged. Checklists and marking schemes can still be used for more traditional tasks with predictable solutions.

Resources

Exploring Geometry With the Geometer's Sketchpad. Key Curriculum Press

The Geometer's Sketchpad™ Archive
www.forum.swarthmore.edu/sketchpad/sketchpad.html
www.forum.swarthmore.edu/sketchpad/gsp.gallery/gallery.html

Graphing Calculator Activities for Enriching Middle School. Texas Instruments

Visualized Geometry: A van Hiele Level Approach. Portland, Maine: J. Weston Walch, 1990.

Activity 3.1: Fence Me In

Time: 150 minutes

Description

In this activity students explore the relationships between perimeter and area of a figure when one of the measures is fixed. They also gain experience using the computer.

Strand(s) and Expectations

Strand(s): Number Sense and Algebra, Relationships, Measurement and Geometry
Specific Expectations: NA 1.06; NA 4.01, .03; RE1.03, .04, .05, .06; MG1.01, .04; MG2.03, .04.

Planning Notes

- This activity requires the students to have a rudimentary knowledge of *The Geometer's Sketchpad*™. Allow 75 minutes to introduce the use of this program prior to the Fence Me In activity. Specifically, students should be familiarized with the toolbar, the names of the tools, and the menus. Resources for this are easily found by conducting a web search using key words "Geometer's
- Photocopy Appendix 3.1 - Fence Me In for each student in the class.
- Reserve the computer lab for this activity.
- While this activity has been designed to meet curriculum expectations regarding the use of technology, students can perform this investigation using grid paper or geoboards in place of *The Geometer's Sketchpad*™. The spreadsheet capability of the graphing calculators could be used for the data analysis.
- Appendix 3.1 has been written for use with *Excel* version 7.0 for Windows 95. Teachers may need to modify the instructions to fit the spreadsheet that they have available or provide appropriate instructions if using graphing calculators. See Building a Garden Fence, *TI 80/82/83 Graphing Calculator Activities for Enriching Middle School Mathematics*.

Prior Knowledge Required

The students should be proficient in creating a spreadsheet.

Teaching/Learning Strategies

Teacher Facilitation: Guide students through a discussion of both problems and the formulation of their hypotheses for each. Adequate time should be given so students can record their hypotheses before they begin their investigations. Ensure that the students have selected the appropriate program settings for *The Geometer's Sketchpad* prior to starting the activity (steps 1 and 2). Circulate among the students and assist as needed. Encourage students to use mental math and estimation in creating their hypotheses and in judging the reasonableness of answers produced by the computer.

Student Activity: Appendix 3.1 - Fence Me In
Students work in pairs.

Follow-Up Measurement Skills: Practise perimeter and area questions involving various shapes from their textbooks.

Assessment/Evaluation Techniques

As students are working through the activity their Knowledge/Understanding skills (using *The Geometer's Sketchpad*™ to collect data and a spreadsheet to organize their data) may be assessed using a checklist. Collect the students' written responses and assess Application skills (the correct analysis of data; identification of applications for maximizing area and minimizing perimeter). Their Communication skills (clarity; justification of reasoning; description of their applications) can also be assessed.

Accommodations

The teacher could assign partners to assist students who encounter difficulties reading instructions or using the computer.

Resources

TI-80/82/83 Explorations. *Graphing Calculator Activities for Enriching Middle School Mathematics.*

Appendix 3.1: Fence Me In Worksheet

This activity uses *The Geometer's Sketchpad*™ and *Excel* to explore different ways of finding the optimal area or perimeter of a rectangular object when the other measure is fixed. You submit the written responses to questions that are asked in this investigation. You will be evaluated on the quality of your work. Your writing should be clear, concise, and accurate and should use appropriate units and correct sentence structure.

A. Finding the Maximum Area

Suppose you had 20 m of fencing and wanted to fence in part of your backyard for a pet rabbit. You want the pen to be rectangular and to have the largest area possible. This activity answers the question, "What is the largest area possible for the rabbit?"

Thinking About the Problem

Answer these questions in your notebook:

1. How are length and width related in this problem?
2. What dimensions do you think give the maximum area? Why are these dimensions reasonable? Explain your thinking.

Setting Up *The Geometer's Sketchpad*

1. Open *The Geometer's Sketchpad*™.
2. Select *Preferences* from the *Display* menu. Set *Distance Unit* to cm and *Distance Unit Precision* to units. The scale for this investigation is 1 cm = 1 m.
3. Choose *Show Grid* from the *Graph* menu. Then choose *Hide Axes* from the *Graph* menu to show only the grid.

The Investigation

4. Using the *straightedge tool*, starting on one of the grid points, construct a rectangle on the grid.
5. Label the vertices using the *text tool*.
6. Highlight the width of the rectangle using the *selection arrow*. Choose *Length* from the *Measure* menu to display the width measurement. Repeat this procedure to display the length measurement.
7. Select all of the vertices by using the *selection arrow* while holding the *Shift* key down. Then choose *Polygon Interior* from the *Construct* menu. You can change the colour of the interior if you like from the *Display* menu.
8. Make sure the interior is selected. Then select *Perimeter* and *Area* of the rectangle from the *Measure* menu.
9. Use the *selection arrow* to highlight one of the sides and then drag the side in or out until the perimeter equals *20.00 cm*. (Be careful not to drag the vertices of the rectangle at any time - this will change its shape. If you do move one of the vertices, select *Undo* from the *Edit* menu, or press *ctrl-z*).
10. Set up a table in your notes for your rough work like this:

Width	Length	Area	Perimeter
			20

11. Copy into the table the width, length and area of your rectangle.
12. Drag the sides of the rectangle until you have a different size rectangle, *but with perimeter still 20.00 cm*. Continue to record different measurements until there are no more possibilities (you should have 9 rows).

Setting Up a Spreadsheet

13. Open *Excel* and create the following worksheet:

	A	B	C	D
1	Finding Maximum Area			
2				
3	Width	Length	Perimeter	Area
4			20	

14. Fill out the worksheet from the table you created in your notes.
15. Select the *Sort* option from the *Data* menu. Click *OK* and the values in the first column will be sorted in ascending order.

Create a Scatterplot Relating Rectangle Width to Area.

16. Highlight cells A4-A12 with the mouse (these are all the width measurements). Then while holding down *ctrl*, use the mouse to highlight cells D4-D12 (the area measurements).
17. With these two columns highlighted, choose *Chart* and *As new sheet* from the *Insert* menu.
18. A *Chart Wizard* box opens. Click *Next.*
19. Select the chart type as *XY-Scatterplot* (centre graph), and click on *Next.*
20. Select 2 from the next box, click on *Next.*
21. Click *Next* again, to get *to Step 5 of 5*. Choose the following options (use *tab* or the mouse to move between options):
    - no legend
    - title the graph "Finding Maximum Area"
    - title the x-axis "Width" and the y-axis "Area"
22. Click *Finish*.

Questions: Continue to answer these in your notebook.

1. What is the maximum area for your rabbit pen? Was your hypothesis correct?
2. Determine a formula that finds:
   (a) the length, and
   (b) the area, if you know the perimeter and the width.
3. Check your formulas with your data in your table. Do they work?
4. Describe how to find the maximum area from:
   (a) a table (spreadsheet);
   (b) a graph.
5. If decimal values were permitted for the length and width would your result be different? Explain how you would use your existing data to justify your response.

B. Finding the Minimum Perimeter

In the first part of this activity you generated data and entered it into a spreadsheet. In this investigation the spreadsheet does much of the work for you.

Suppose you had some seeds to plant a vegetable garden. Following the directions on the packets for spacing the seed you realize the area of the garden is to be 100 m^2. You want to put up a decorative wooden lattice around the rectangular garden. Since the wood is expensive you would like to design the garden so that you have to buy the least amount of lattice possible (the smallest perimeter). In this investigation you will find the minimum perimeter for an area of 100 m^2.

Getting Ready: Answer these in your notebook.

1. What do you think is the minimum amount of lattice needed? Explain your thinking.
2. If the width of a rectangle is w metres and its area is A metres squared create an algebraic expression for the length using w and A.
3. If the length of the same rectangle is l create an algebraic expression for the perimeter using l and w.

Setting Up a Spreadsheet

1. Open *Excel*, and create the following worksheet:

	A	B	C	D
1	Finding Minimum Perimeter			
2				
3	Width	Length	Perimeter	Area
4	2	= D4/A4	= 2*A4 + 2*B4	100
5	= A4+2			

Note: Hit *Enter* after entering each formula above. The value of the formula appears in the cell.

2. Highlight A5 and while still holding the mouse button down, drag the mouse until row 50 is also highlighted. Release the mouse button and select *Fill* and *Down* from the *Edit* menu. Notice the values that have been entered in column A.
3. Highlight B4, C4, and D4 and while still holding the mouse button down, drag the mouse until row 50 is also highlighted. Select *Fill* and *Down* from the *Edit* menu. Notice the values that have been entered in columns B, C, and D.

Create a Scatterplot Relating Rectangle Width and Perimeter

4. Create a chart on a new sheet (as in part A), labelling the title and axes appropriately.

Questions

Continue to answer these in your notebook.

1. What is the minimum amount of lattice that you need to buy? Was your hypothesis correct?
2. Explain why we would want to calculate maximum area when the perimeter is fixed or a minimum perimeter when the area is fixed.
3. What are some real-life examples of situations where it might be necessary to find the maximum area with a fixed perimeter or the minimum perimeter with a fixed area?

Challenge Question

4. Could an area of 100 m^2 be enclosed by a smaller perimeter if the shape did not have to be a rectangle? Explain your thinking.

Activity 3.2: The Size Is Right!

Time: 150 minutes

Description

In this activity each student pair works with an open-topped, square-based prism and a cylindrical container, and fills each one with the same volume of material. They then measure the depth of the fill in each and imagine the depth of fill as the height of the container. They then calculate the surface area of each container. Class results are gathered so that relationships between volume and surface area can be analysed. In the Follow-up Activities students develop, through investigation, formulas for the surface area and the volume of cones, pyramids, and spheres.

Strand(s) and Expectations

Strand(s): Number Sense and Algebra, Relationships, Measurement and Geometry

Specific Expectations: NA1.02, .04, 1.06; NA 2.02; NA3.04, .05; NA4.01, .02, .03; NA5.03; RE1.03, .04, .06, .07; MG1.01, .02, .03; MG2.01, .02, .03, .04.

Planning Notes:

The class gathers a large variety of cylindrical containers, each having a volume of at least 250 mL. Students use stiff construction paper to build a square-based prism for homework the night before this activity is to be done in class.

Prior Knowledge Required

- Knowledge of the distinguishing characteristics of a square-based prism and of a cylinder as well as how to calculate the volume and surface area of each

Teaching/Learning Strategies

Teacher Facilitation: Establish student pairs for this activity in the previous class.

Give pairs about five minutes of time working together at the end of the previous class to plan a model for an open-topped, square-based prism which has a volume of between 300 and 500 mL. For homework, one student in each pair constructs the model, while the other student draws the net for the model and computes the surface area of a closed container like their model.

The teacher should have a couple of appropriate square-based prisms (e.g., one cracker box could be cut into two prisms) handy in case a model-building partner is away ill or forgets to bring the model to class.

Provide each pair of students with a different radius cylindrical container having a volume between 250 and 500 mL (e.g., cans for soup, tomato paste, drinking glasses, shampoo bottles).

Provide each pair of students with exactly 250 mL of material that can be poured into the different containers (e.g., small beads, rice, Styrofoam pellets, or similarly small-sized, hard, dry material) Each pair needs a ruler marked in millimetres.

After all class data has been entered, lead the discussion about minimum surface area. The degree of teacher direction depends on the class and on the variety of containers used.

Interpolation may be needed to answer questions #6 and #7, if no pair built the square-based prism that results in a cube when holding 250 mL, or if the teacher did not provide the cylinder that results in height equal to diameter.

Student Activity:
Student pairs should carry out the following:

1. Check your partner's homework on the square-based prism. Discuss and fix any problems or discrepancies.
2. Pour the 250 mL of material into your square-based prism. Shake it gently to ensure that the top surface is flat. Measure the height to which your material fills the prism. Record this height in the table below.
3. Pour your material into the cylindrical container provided. Shake it gently to ensure that the top surface is flat. Measure the height to which your material fills the cylinder. Record this height in the table below.
4. Complete the rest of the table. Record height measurements accurate to the nearest millimetres. Use the volume of fill and the volume formula for each shape to compute the base length or radius measures for each shape. Verify these calculations by measuring the actual containers with a ruler. Show all calculations.

   Note: *When calculating the surface area for a closed container, imagine the container that uses the bottom and side walls of the actual container you poured your material into, but has a top right on top of the upper surface of your material. This closed container would have a volume of exactly 250 mL.

Height in Square-based Prism	Length and Width of Prism Base	Surface Area for a Closed Prism*	Height in Cylinder	Radius of Cylinder	Surface Area of Closed Cylinder*

5. Enter your final measures only on the class summary.
6. By referring to the class data, what is the minimum surface area for a closed square-based prism that has a volume of 250 mL? What are the dimensions of this prism?
7. Graph the data for the square-based prism from the table by setting height as the independent variable and surface area as the dependent variable. Is this relation linear? Does your graph help you to confirm your answers to #6? Explain.
8. By referring to the class data, what is the minimum surface area for a closed cylinder that has a volume of 250 mL? What are the dimensions of this cylinder?
9. Describe what is special about the shapes of the 250 mL containers that have minimal surface area? Justify your reasoning.
10. Describe situations in which it is important to know the minimum surface area for a given volume.

Teacher Facilitation: Encourage pairs of students to check each other's work. Observe the accuracy with which students are working and give mini-lessons as needed. As students work with substitution and evaluation of formulas, they may need reminders about the order of operations, the

Manage the accumulation and displaying of data from all pairs of students. This could be done using a spreadsheet and projected for the class, or in a table on the blackboard.
Circulate among the students and use a colourful pen to prompt appropriate thinking on questions 6-9, as needed.

Follow-up Activity 1:

1. Construct a cone having the same radius and height as your 250 mL cylinder.
2. Hypothesize how the volume of this cone compares to the volume of the cylinder.

3. Fill this cone as many times as you can until you have used your 250 mL of material. Does this confirm or deny your hypothesis?
4. What would be an appropriate formula for the volume of a cone having radius r and height h?
5. Determine what the results would be if the calculations were done for open-topped containers.
6. If each dimension of your square-based prism were doubled, what would be the effect on:
   a) the surface area? b) the volume?
7. What would be the effect of tripling each measure for your prism?
8. If the radius and height of your cylinder were doubled, what would be the effect on:
   a) the surface area? b) the volume?
9. What would be the effect of tripling each measure for your cylinder?
10. Summarize your findings.

Follow-up Activity 2:

Teacher Facilitation: Ensure that the results of Follow-up Activity 1 are summarized so that the students have developed the formulas correctly and that they can identify the effect of varying the dimensions of a prism or cylinder on the volume or surface area of the object.

Draw from the students the fact that the volume of 3-D prisms of any shape and cylinders, is the base area times the height. This is a very useful concept because it saves memorization.

Through investigation, students develop the formulas for volume and surface area of the pyramid and the sphere using any of the techniques outlined in the new grade nine textbooks. The teacher has to be sure that the equipment and materials required by the activity are available for the students and that time is set aside to summarize the results of the investigations.

Assign textbook work so that students practise using the formulas, solve problems that require the formulas, and answer questions using either exact or approximate answers, depending on the context.

Assessment/Evaluation Techniques

- Students could hand in a "log of problem solving ideas" that lead them to their discoveries. This could be assessed using a rubric focussing on Thinking/Inquiry and Communication.
- Students could design a similar problem in a real-world context, solve the problem, and present the problem and solution to the class. This could be peer-assessed for Communication and teacher-assessed for Application of Knowledge.
- Students could be given a quiz to ensure that they correctly select and use the appropriate formulas to solve problems

Accommodations

Use any of the following accommodations: structure extensions; increase time lines; walk student through "small chunks" of tasks; provide both written and oral instructions; help student organize what needs to be done first - How To Get Started; help students to categorize and organize the formulas as they work with them.

Activity 3.3: Why Do Elephants Have Big Ears?

Time: 150 minutes

Description

In this activity, students investigate volume and surface area of square prisms in order to explain the significance of optimal surface area and volume to heat loss. They gather and analyse data to model the relationship between an elephant's surface area and volume. They form ratios that will lead them to answer the question "Why do elephants have large ears?", as well as other relevant questions such as "Why shouldn't dogs or babies be left in cars in the summer heat?" Students also investigate the relationship between surface area and volume of rectangular prisms.

Strand(s) and Expectations

Strand(s): Measurement and Geometry, Relationships, Number Sense and Algebra
Specific Expectations: MG1.01, .02, .03, MG2.01, .02, .03, RE1.01, .04, .05, .06, .07, NA1.01, .03, .04, .05.

Planning Notes

- Copies of the student worksheet Why Do Elephants Have Big Ears? (from *Math Mania* magazine) must be copied or downloaded from the web site given in the Resources. Alternatively, the teacher creates a worksheet that has students examine the relationship between surface area and volume of a cube as the dimensions are varied.
- Make copies of the Can You Do Better worksheet available for each student.
- Students use a graphing calculator or spreadsheet to do the Can You Do Better activity.

Prior Learning Required

- Ratio and rates
- Formulas for surface area and volume
- Using a spreadsheet or graphing calculator

Teaching/Learning Strategies

Teacher Facilitation: Model the size and shape of an elephant using a cube to represent the shape of an elephant. Discuss the setup of the surface area to volume ratio that is examined. Prompt the students as they complete the activity.

Student Activity: Complete the Why Do Elephants Have Big Ears investigation.

Follow-up Activity

Teacher Facilitation: Students can use spreadsheets or the LISTS functions on a graphing calculator to calculate their data for the chart below. If using LISTS, students may need assistance to enter *l* in L_1 as a sequence using LIST,OPS,seq(A, A, 1, 8, 1), *w* in L_2 using the formula "64/(4xL_1)", and surface area in L_3 as "2xL_1xL_2+8x(L_1+L_2)". They do a scatterplot graph on L_1 and L_3. Whole class discussion is needed after students have answered Part A to confirm their findings. Before the students start Part B they are to be assigned a different fixed height (1 to 8). Ensure each fixed height is done by at least two students.

Student Activity: Can You Do Better?
Students fill in a chart similar to the one given below, and answer the questions.
Part A: Using a fixed volume of 64 cm^3, calculate each surface area by varying the dimensions of the base where the height is fixed at 4 cm.

V	*h*	*l*	*w*	**Surface Area**
64	4	1		
64	4	2		
. .	. .	. .		
64	4	8		

1. Examine the values for the surface area. What is the relationship between the surface area and the various lengths?
2. Graph the relationship between length and surface area.
3. For which length is the surface area the smallest? Why might this be important?
4. Is there anything special about this dimension?

Part B. Can you do better if you changed the fixed height?

1. From your teacher obtain your new fixed height to investigate. Redo the calculations and questions in Part A.
2. What was your smallest surface area? Compare it to the optimal surface area in Part A. Did you do better? (i.e., Was the surface area smaller?)
3. Compare your results to the results from other students.
4. Make a hypothesis about the dimensions of a rectangular prism and optimal surface area.

Part C

1. The Big Chill Drink Company has hired you to design a new drink container to hold 216 ml. What dimensions would you suggest they use? Justify your answer. Is it realistic? Why or why not.
2. Calculate the surface area of an ordinary rectangular juice carton. If you re-design the container to have a minimum surface area but hold the same volume of juice, what would be the percentage decrease in material required?

Homework:

- Other optimization problems involving surface area and volume of other shapes as found in the textbook
- Ratio and rate problems

Assessment/Evaluation Techniques

Use a rubric to observe the students using their Inquiry and Thinking skills. A written report for Can You Do Better? can also be evaluated using a rubric to assess Knowledge, Application, and Communication skills. Responses to Part C lend themselves to the possibility of Level 4 if part of the student's answer is "Realistically, a cube is not the best shape because..." Paper and pencil tasks can be evaluated as well.

Resources

Math Mania Magazine (February 1999)

Math Mania website
http://www.mathadventures.com

Activity 3.4: Parks and Recreation - Assessment Activity

Time: 150 minutes

Description

In this activity students use their knowledge of area, perimeter, surface area, and volume formulas to design a wading pool and a skateboarding area.

Strand(s) and Expectations

Strand(s): Measurement and Geometry, Number Sense and Algebra

Specific Expectations: MG1.01, MG 1.02, MG1.03, MG1.04, MG2.01, MG2.02, MG2.03, MG2.04; NA 1.01, .02, .03, .04, .05, .06; NA 2.04.

Planning Notes

Students work in pairs or groups of three. Supply materials such as ruler, metre sticks, markers, and graph and chart paper.

Prior Learning Required

Students are to be familiar with perimeter, area, volume and surface area formulas. They also make effective use of a scientific calculator to evaluate formulas after substitution.

Teaching/Learning Strategies

Student Activity: Parks and Recreation

The City Parks Department has decided to put wading pools into each of their parks so that young children are able to stay cool in the summer. They have decided to run a contest for secondary school students to submit designs for these pools. The following are their specifications:

The Wading Pool

Please submit two designs - one is a circular pool and the other is a square pool.

- The depth of the water at the edges of the pool must be between 10-25 cm and then slope to 50 cm in the centre.
- The volume of the water in the pool must be approximately $20m^3$.
- There will also be a concrete walkway around the pool that must be 90 cm wide and 4 cm thick.

Your submission must include:

- detailed drawings of the pools and the walkway, including dimensions;
- calculations of the volume of each pool to demonstrate that the volume is 20 m^3;
- calculations of the surface area of each of the pools so that the finance department can calculate the cost of a pool liner or sealer. A pool sealer costs about $\$22.50/m^2$ to have made.
- calculations of the volume of concrete required for the walkway for each of the pools.

The Parks and Recreation department received several phone calls from secondary school students when the call for the pool designs went out. They were outraged that young children were getting new wading pools when the secondary school students had been constantly requesting skateboarding areas to be built in parks. The Parks and Recreation department still wanted to proceed with the wading pools but conceded that they would set aside some money for the skate boarding areas. They were fortunate to have a local building company donate 100 m of fencing to enclose each skateboarding area. Again, they called for designs to be submitted from secondary school students. Here are the specifications:

Skateboarding Area

Design a skateboarding area that has an irregular shape composed of at least three of these five shapes: rectangles, circle, semi-circle, trapezoid, and parallelogram. This area must be able to be enclosed within a rectangular fence using 100 m of fencing. Your submission should include:

- a scale drawing of the skate boarding area with the rectangular fence;
- calculations of the area of the skateboarding surface so that the Finance department can calculate the cost of painting the “blacktop” every year. The cost is $\$4.25/m^2$ plus GST and PST.

Teacher Facilitation: The teacher should circulate as students work, prompt when necessary and collect observational assessment data.

Assessment/Evaluation Techniques

While students are working on this activity, teachers could be circulating and gathering observational data on Learning Skills such as: teamwork, organization, and initiative.

Problem Solving/Inquiry skills could be assessed using a rubric.

Once the activity is completed, groups could trade their submissions with a "Buddy Group" and check one another's calculations. Each group could then present their plans to the class or put them on display for peer assessment. A checklist could be used for this peer assessment which would include:

- accuracy of calculations (checked by "buddy group") of volume, surface area, cost of pool sealer, volume of concrete for walkway, area of skateboard section, cost of painting "blacktop";
- clear, well-labelled drawings and diagrams;
- incorporation of required shapes in skateboard design;
- maintenance of required volume of approximately 20 m^3 for pool and perimeter of 100 m for skateboard area.

Review and compile the peer assessments and include them in the overall assessment of the activity.

Activity 3.5: Exploring with *The Geometer's Sketchpad*™

Time: 150 minutes

Description:

In this activity, students explore and review angle relationships as they become familiar with dynamic geometry software. Using the Angle Relationships student worksheet, students sketch angles relationships, make hypotheses about the relationships, and verify or refute the hypotheses using dynamic geometry software. This worksheet also places the angle work in contexts such as the design of bridges and roads, the path of a billiard ball, and Mexican blanket designs to demonstrate the application of geometric relationships.

Strand(s) and Expectations

Strand(s): Measurement and Geometry

Specific Expectations: MG3.01, MG3.04, MG3.05.

Planning Notes

- A computer lab is required to complete this exploration. Be familiar with the software so that technical difficulties can be anticipated.
- This activity is best done individually or in pairs, with each student maintaining an individual journal of diagrams and angle relationships.

Prior Knowledge Required

Geometry and Spatial Sense Grade 8: identify the angle properties of intersecting, parallel and perpendicular lines by direct measurement (interior, corresponding, opposite, supplementary, and complementary angles); explore the relationship to each other of the internal angles in a triangle; solve angle measurement problems involving properties of intersecting line segments, parallel lines and transversals; create and solve angle measurement problems for triangles.

Teaching/Learning Strategies

Discuss the following uses of mathematics in career situations with the class, in order to build a context for the exploration:

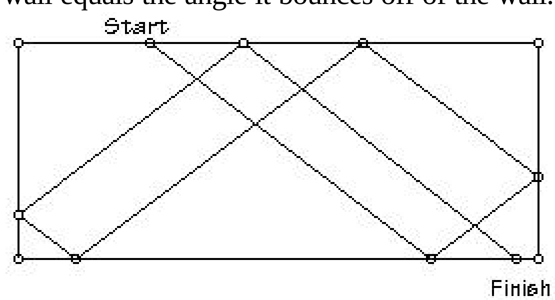
A civil engineer designs bridges, buildings, and networks of roads. A draftsman makes scale drawings of the designs. Angles and triangles are very important to both the engineer and the draftsman as they model situations for the structures that they are designing. The special properties and relationships found in triangles and angles help in the design and stability of visually appealing buildings and safe roads. Computer Assisted Design (CAD) programs are used to draw geometrically exact drawings. We use *The Geometer's Sketchpad*™ to construct models that mimic some of the engineering design work as we review angle relationships.

> **Teacher Facilitation:** Show students aerial photographs of road patterns, maps of roads, and buildings with intricate geometric designs as you discuss the use of angles and shapes in construction. Discuss drafting and design as done with and without the aid of computers. Demonstrate some of the capabilities of *The Geometer's Sketchpad*™ for your students or simply let them investigate and learn the program on their own, under your guidance.

Student Activity: Students use the dynamic geometry software and the Angle Relationships worksheet to explore angle relationships, form hypotheses, verify or refute the hypotheses and communicate the results.

Extension:

1. Draw the path of a billiard ball on a pool table for 6 bounces off the side of the table. Remember that the angle the ball hits the wall equals the angle it bounces off of the wall.



2. Draw a map using angle relationships and skills from *The Geometer's Sketchpad*™. Include as many different angle relationships as possible.
3. Many different cultures use geometric designs in their crafts. Find examples of Mexican blanket designs, Native Canadian beadwork and basket work, etc., and make a geometric design similar to one of them using a variety of different angle and triangle relationships.
4. Determine angle measures in multi-step problems from diagrams found in the textbook.

Assessment/Evaluation Techniques

- Teacher Observation of Students at Work in Problem Solving (see rubric in Appendix)
- Written report on angle relationships
- Paper and pencil assessment tasks determining angle measures in individual and multi-step problems

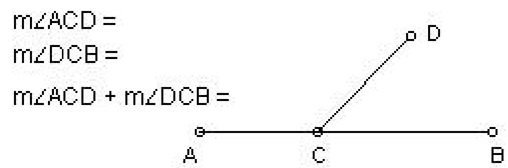
Resources

Exploring Geometry with The Geometer's Sketchpad. Key Curriculum Press.

Appendix: Angle Relationships Student Worksheet

Use *The Geometer's Sketchpad*™ to draw each angle relationship below. Make a conjecture about each type of angle. Use *Sketchpad*'s calculator to confirm or deny your conjecture, then test by dragging points on the diagram. Copy each diagram into your notebook and complete a statement about the angle relationship. (Some construction hints are provided.)

Supplementary Angles:

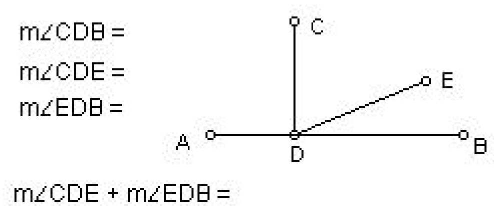


Make a hypothesis about the relationship between the 2 supplementary angles, ∠DCB and ∠ACD.

Check your hypothesis by using *The Geometer's Sketchpad*™ *calculator,* following this method: From the *Measure* menu, choose *Calculate* and a calculator appears on the screen. Click on the workspace where you see m ∠DCB and it will appear on the calculator screen. Click the + sign, and then click the work space where m ∠ACD appears. The calculator should now display the sum m ∠DCB + m ∠ACD. Click *OK*. You return to the workspace and the sum will be calculated on the screen for you.

Test your hypothesis in a variety of different cases by dragging the point D to the right and left. Watch the angle measurements change as you drag the point. Examine the sum as the angle sizes change.

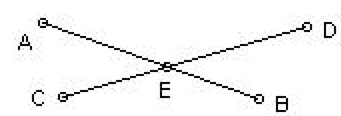
Complementary Angles:



Make and check a hypothesis about the complementary angles, ∠CDE and ∠EDB.

To construct the perpendicular line, select point C and line AB. Choose *Perpendicular line* from the *Construct* menu. Use the point tool to place a point at D. Drag point E to test all cases of your hypothesis.

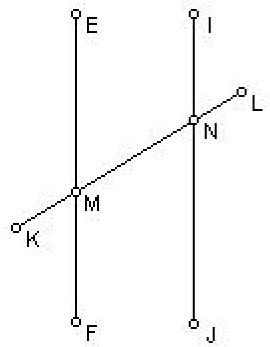
Vertically Opposite Angles:



Use the line segment tool to draw two intersecting lines. Select both lines. From the *Construct menu,* choose *Point of Intersection*.

Make a conjecture and then measure all 4 angles to verify the conjecture. Drag point D to test your conjecture for all cases.

Angle Relationships in Parallel Lines:



When houses and other buildings are built, parallel beams are used to support the building. The crossbars or transversals provide strength to the vertical beams. Three special angle relationships exist within parallel lines. Let's investigate!

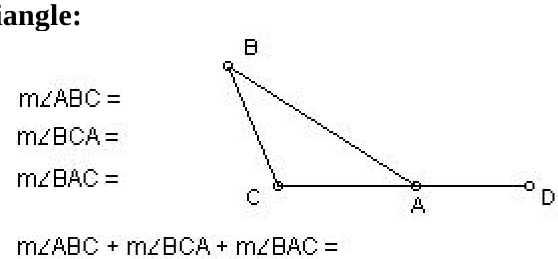
To draw parallel lines, select both a point and a line and choose *Parallel Line* from the *Construct* menu. Draw a transversal KL.

Write a hypothesis about a pair of angles that might be equal. Verify your hypothesis by measuring the angles and then dragging the transversal to ensure the accuracy of your hypothesis.

Can you hypothesize and verify 3 different angle patterns? Hint: The angles appear in patterns that resemble F (corresponding angles), Z (alternate angles) and C (interior angles).

An easier way to make parallel lines is by using transformations: Drag a line. From the *Transform* menu, select *Translate*. Click *OK*. (You may alter the magnitude if you wish.)

Sum of the Angles in a Triangle:



Write a hypothesis about the sum of the angle measures of the triangle.

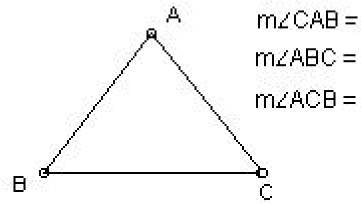
Verify your hypothesis by determining the measure of each angle of the triangle and using *Sketchpad*'s calculator.

Drag a vertex of the triangle and watch the angle measures change. Does the sum change?

Exterior Angle of a Triangle:

Measure the external angle ∠BAD above and compare this measure to the sum of the measures of the two interior angles of the triangle that are not adjacent to ∠BAD. Make a hypothesis about the exterior angle of a triangle. Test the hypothesis by dragging various points in the diagram.

Angles of an Isosceles Triangle:



To construct an isosceles triangle, draw a line segment AB. Click on point A. From the *Transform* menu, *Mark Center "A"* as the center of rotation. To rotate the line segment AB, select AB. From the *Transform* menu, select *Rotat*e and choose an angle of 80°. A rotated segment appears. Place a point at C and then add the third side of the triangle.

Make a hypothesis about the angles in the isosceles triangle. Measure, drag vertices, and verify your hypothesis.

If you were to draw an equilateral triangle, what angle of rotation must be used? Make an equilateral triangle, and check your accuracy by measuring all sides and angles.

There are three different ways to construct an isosceles triangle:

1. Using rotations
2. Using reflections (under the *Construct* menu).
3. Using a circle. (Remember that radii of a circle are equal. When you draw the triangle inside the circle, use *Hide Circle* from the *Display* menu.)

Construct an isosceles triangle using each of the methods listed above. Determine situations for which each method is best suited and least suited.

Activity 3.6: If ... Then ...

Time: 300 Minutes

Description

Using the worksheets provided in the Appendix, students develop and extend "If... then..." hypotheses for a variety of geometric figures. To do this, they explore geometric properties, form an hypothesis related to their exploration, test the hypothesis using dynamic geometry software, generalize the results and communicate the results using "If... then..." format.

The specific topics of the student investigation worksheets are:

- Investigating Midpoints and Medians
- Investigating Perpendicular Bisectors
- Investigating Angle Bisectors
- Investigating Polygons

Strand(s) and Expectations

Strand(s): Measurement and Geometry
Specific Expectations: MG3.01, .02, .03, .04, .05.

Planning Notes

- A context should be provided for these activities through a variety of different methods:
  a) Complete a variety of paper folding activities that lead to hypotheses about geometric properties that can be verified using geometry software.
  b) Provide a pictorial display of geometrically-based shapes and designs from the world around us. Through discussion, develop curiosity about geometric characteristics and relationships in the shapes: Are the lines the same length? Are the angles equal? Do they always intersect? Are they bisected? What if ...?
  c) Make paper and pencil constructions using compass and protractor. Form hypotheses as well as illustrate the accuracy, time, and skill required in comparison to the ease of geometry software.
  d) Use MIRA reflections to formulate hypotheses and refute or verify using geometry software.
- A computer lab with dynamic geometry software is required to complete the activities. Be familiar with the geometry software that technical difficulties can be anticipated.
- These activities are best done in pairs, with each student maintaining an individual journal of diagrams, hypotheses and generalizations in the "If.., then..." form.
- Homework assigned each day could be explorations using paper-folding, compass/protractor or MIRAs to develop hypotheses which they investigate the following day using dynamic geometry software. At the end of each computer lab session, some journal writing and paper/pencil tasks could be assigned from the textbook.

Prior Learning Required

Geometry and Spatial Sense Grade 7 and 8: Identify, describe, compare, and classify geometric figures. Identify congruent and similar figures. Identify and investigate the relationships of angles. Construct and solve problems involving lines and angles.

Teaching/Learning Strategies

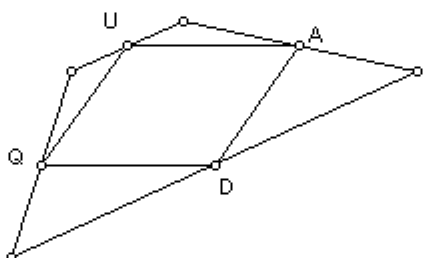
One sample activity, Midpoints of Quadrilaterals, found in the investigations provided on the student worksheets in the Appendix, is examined in detail here, to model the teacher set-up and facilitation that can be followed for the other investigations. The steps *Explore*, *Form a Hypothesis*, *Test the Hypothesis*, and *Communicate the Results* should be an integral part of each investigation. Students could pair/share their hypotheses and homework exploration results before investigating on the computer. Ensure closure and consolidation of newly developed concepts by leading class discussions on the concepts after the investigation.

Teacher Facilitation: (This is a sample introduction to lead to the investigation of midpoints of a quadrilateral on the student worksheet in the Appendix.) Use a paper cutter to create a variety of quadrilaterals to distribute to the students. Draw to the students' attention the fact that they each have different quadrilaterals.

Explore:
Hold two adjacent vertices of your quadrilateral, one in each hand. Bring your hands together so that the vertices are together. Crimp the paper at the midpoint between the vertices. Repeat step 1 for each pair of adjacent vertices, until all four midpoints have been located. Fold your paper between adjacent midpoints. You should have four such folds, creating a quadrilateral. Label this quadrilateral QUAD.

Form a Hypothesis: Make a hypothesis about the quadrilateral QUAD.

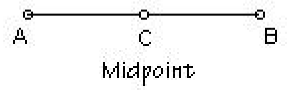
Test Your Hypothesis:
Identify measurements that will be needed to confirm or refute your hypothesis.
Using *The Geometer's Sketchpad*™, draw any quadrilateral as illustrated below. Construct *Point at Midpoint* on each of the four sides. Label the midpoints, in order, Q, U, A, and D. Construct a segment between adjacent midpoints to create quadrilateral QUAD. Do a drag test of your construction.



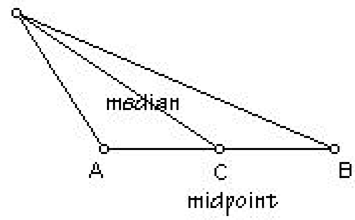
Length(Segment DQ)	0.89	
Length(Segment AD)	0.76	
Length(Segment UA)	0.89	
Length(Segment QU)	0.76	

Slope(Segment DQ)	-0.08		
Slope(Segment QU)	1.37		
Slope(Segment AD)	1.37		
Slope(Segment UA)	-0.08		

Appendix: Investigating Midpoints and Medians



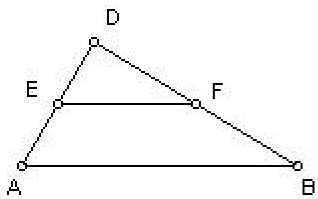
A **midpoint** of a line segment is a *point* that divides a line segment into two equal parts. To construct the midpoint of a line segment, select the line and choose *Point at Midpoint* from the *Construct* menu.



A **median** is a *line* that joins a midpoint to the vertex directly across from the midpoint. To construct a median, determine the midpoint first, and then draw the median, connecting the midpoint to the opposite vertex.

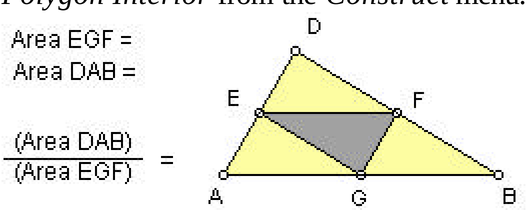
In the following activities, we investigate midpoints and medians.

Midpoint Lines of a Triangle:
Draw a scalene triangle and construct the midpoints of two of the sides. Join the midpoints with a line.



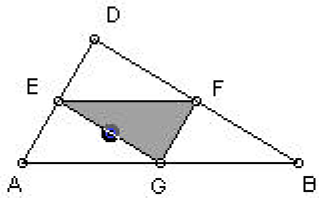
a) Examine the angles created where the midpoint line intersects the two sides of the triangle. Compare these angles to the angles at the base of the triangle.
   Measure the length of the midpoint line and compare it to the length of the third side of the triangle.
b) Draw an appropriate diagram in your notebook to illustrate the statement "If a line joins the midpoints

c) Determine the third midpoint. Construct the midpoint triangle by joining the midpoints. Make a hypothesis about the ratio of the area of the midpoint triangle to the area of the entire triangle.
   To measure the area of the midpoint triangle, select all three vertices of the triangle and choose *Polygon Interior* from the *Construct* menu.



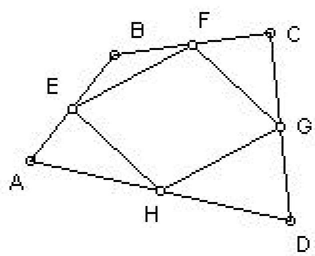
   When the interior is shaded, its area can be determined by choosing *Area* from the *Measure* menu.
   Verify your hypothesis about the ratio of the areas of the midpoint triangle and the original triangle.

d) Prove that the midpoint triangle is congruent to $\triangle$EAG and $\triangle$DEF and $\triangle$FGB using rotations: Determine the midpoint of line segment EG. Set this midpoint as the center of rotation by choosing *Mark as Center* from the *Transform* menu. Select the midpoint triangle and determine the angle of rotation that will place it directly on top of $\triangle$EAG, thereby proving that the two triangles are congruent. (Use the *Undo* command found under the *Edit* menu if you make an error.) Find other midpoints and rotations to prove the remaining triangles congruent to the midpoint triangle. Explain your answer to part c) above using what you just discovered about the congruent triangles.



Midpoints of Quadrilaterals:

a) Draw a quadrilateral. Determine the midpoints of each of the 4 sides. Join the midpoints in order to construct another quadrilateral. Make a conjecture about the type of quadrilateral formed by joining the midpoints. Verify your conjecture. Complete and illustrate the statement: “If the midpoints of a

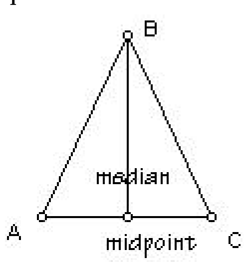


b) Determine if there is a relationship between the areas of the midpoint quadrilateral and the original quadrilateral. If a relationship exists, write an “If...then...” statement about the relationship.

c) Repeat the midpoint quadrilateral investigation but alter the original quadrilateral so that it is a:
1) parallelogram 2) a rectangle 3) a square 4) a rhombus.
Make “If... then...” statements for each case and draw the corresponding diagram in your notebook.

Median of an Isosceles Triangle:

Construct an isosceles triangle. Determine the midpoint of the base of the isosceles triangle. Draw the median by joining the midpoint to the vertex of the vertical angle. Conjecture and verify if any angle equalities exist in the diagram.



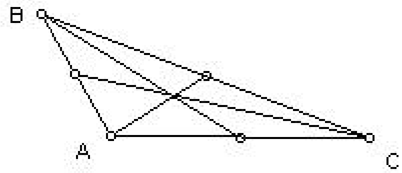
Complete the statement: “If the median to the base of an isosceles triangle is drawn, then...”

Explain why the safest position for a step ladder is standing spread open on flat ground.

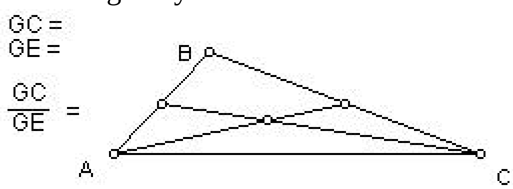
Describe other situations where isosceles triangles, midpoints, and medians are used in the world around us.

Medians of a Triangle:

a) Draw a scalene triangle. Construct the medians to each of the three sides. The medians intersect at a point called the centroid. The centroid is the center of area and volume, and acts as the centre of gravity of the triangle.



b) Print out the triangle with the centroid located. If you cut out the triangle and glue it to a piece of cardboard, you will be able to balance it on the tip of a pencil using the centroid's property as the center of gravity.



c) The centroid (G) divides each median into a longer and a shorter segment. Make a conjecture comparing the length of the shorter segment to the longer segment. How many smaller segments will fit into the larger segment? Verify your prediction. Write the relationship as a ratio.

Appendix: Investigating Perpendicular Bisectors

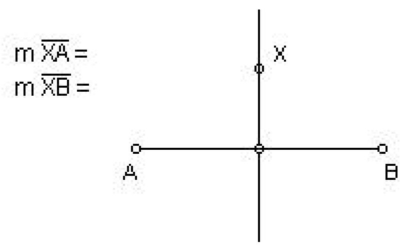
A **perpendicular** is a line that intersects another line segment at a 90° angle.

Given a line segment and a point not on the line, construct a perpendicular to the line from the point. (Select both the point and the line. Choose *Perpendicular* from the *Construct* menu.)

If the perpendicular passes through the midpoint of the line, it is called the perpendicular bisector. To construct the perpendicular bisector of a line, determine the midpoint and construct the perpendicular through it. Measure to ensure that the line segments are equal and the angles measure 90°. In your notebook, illustrate and define a perpendicular line and a perpendicular bisector.

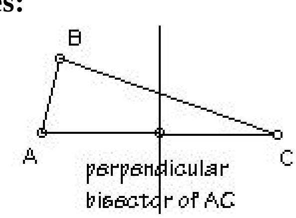
We will now investigate several useful properties of the perpendicular bisector.

A Point Located on the Perpendicular Bisector:
Construct a point X anywhere on the perpendicular bisector. Construct line segments AX and BX to measure the distance from the point X to the endpoints of the line segment. Does a relationship occur between the lengths of AX and BX? Make and verify a conjecture. Test your conjecture by dragging X along the perpendicular bisector, and examining what occurs as the lengths of AX and BX change.



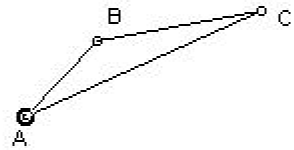
In your notebook, draw a diagram and complete the following statement: "If a point lies on the perpendicular bisector of a line segment, then..."

Perpendicular Bisectors in Triangles:



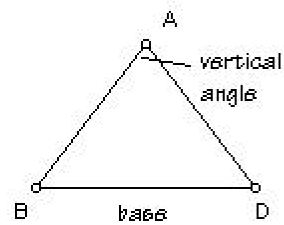
a) Draw an acute triangle ABC. Construct the perpendicular bisectors of each of the sides BC, AB and AC. What do you notice?
b) Name the point of intersection O. Construct a circle, center at O, and radius OA. The circle should also have radii OB and OC too! The point O is called the circumcenter of the triangle.
c) Explain why O is equidistant from the three points A, B, and C. Include your conclusions from the previous question in your explanation.

d) Repeat the investigation for an obtuse-angled triangle. Are the results the same as for the previous investigations?



e) Repeat for a right-angled triangle. What interesting phenomenon occurred with the circumcenter?
f) A fire hydrant must be located the same distance from three different houses in a subdivision. Explain how you would determine the location of the hydrant.
g) Describe other situations where you would use the circumcenter to solve a problem.

The Perpendicular Bisector of the Base of an Isosceles Triangle:
Construct an isosceles triangle △ABD. Construct the perpendicular bisector of the base.



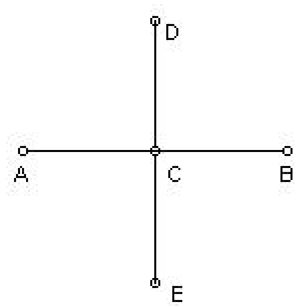
If the perpendicular bisector of the base of an isosceles triangle is drawn, then it will pass through the vertical angle and bisect the vertical angle.

What would you measure to verify the statement? Measure and verify.

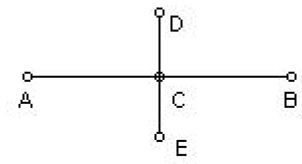
In your notebook, draw an appropriate diagram and label all of the equal measures on the diagram.

For what other types of triangles is this statement true? Construct a right-angled triangle and one other triangle for which the statement is true.

Perpendicular Bisectors as Diagonals of Quadrilaterals:
Draw a line segment AB. Determine C, the midpoint of AB. Mark C as the center of rotation and rotate the line AB 90° using the *Transform* menu. The new line DE is the perpendicular bisector of AB, and, AB is the perpendicular bisector of DE. Join the 4 endpoints to form a quadrilateral AEBD with diagonals AB and DE. Make a hypothesis about the type of quadrilateral formed by the two diagonals. Verify your hypothesis. In your notebook, draw an appropriate diagram and complete this statement: "If the diagonals of a quadrilateral are equal in length and perpendicular bisectors of each other, then..."

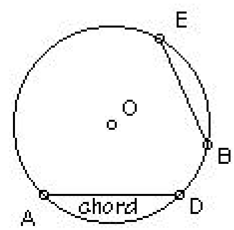


If the diagonals of the quadrilateral are perpendicular bisectors of each other, but the diagonals are not equal in length, what type of quadrilateral is formed? Make a hypothesis, verify your hypothesis, and place a diagram and statement in your notebook.



Perpendicular Bisectors in Circles:

Construct a circle. Draw a chord in the circle. Construct the perpendicular bisector of the chord. Examine the diagram and make a hypothesis about the perpendicular bisector of a chord. Construct a second chord to verify your hypothesis.



Complete this statement: "If the perpendicular bisectors of 2 chords of a circle are drawn, then...". Find a situation for which this statement is not true for two chords of a circle.

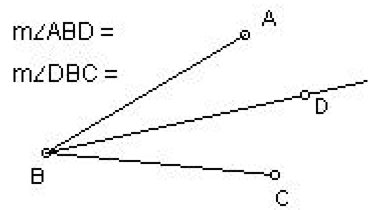
If you were given a circle cut out of paper, explain how you would find the centre of the circle by paper folding.

A carpenter has cut a circular table top for a pedestal (one-legged) table. Explain how the centre of the tabletop can be determined in order to attach the leg.

Describe a realistic situation where you would need to find the centre of a circle.

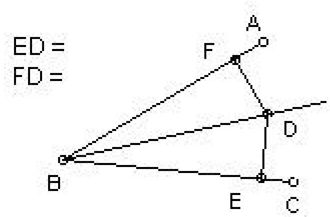
Appendix: Investigating Angle Bisectors

An **angle bisector** is a line that divides an angle into two equal parts. To bisect an angle ABC, select the vertices in order and then choose *Angle Bisector* from the *Construct* menu. Verify that two equal angles were constructed, by measuring each angle.



The following activities provide opportunities investigate bisectors of angles, while making and verifying hypotheses.

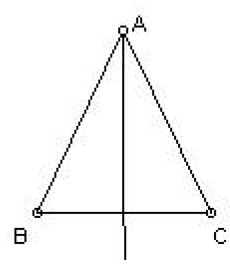
A Point on the Angle Bisector:
Construct the angle bisector of an acute angle. Place a point D on the angle bisector. To measure the distance from the point to the line BC, construct a perpendicular from D to the line. (The shortest distance is always the perpendicular distance) Measure this distance. Determine the distance from D to the other arm of the angle.



Make and test a hypothesis about the distance from any point on the angle bisector to the arms of the angle. Test your hypothesis by dragging the point D along the angle bisector.

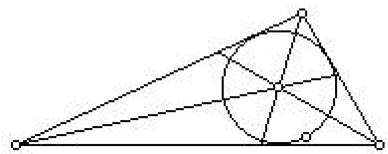
Test your hypothesis on obtuse, right, and straight angles. Draw an appropriate diagram in your notebook and complete the statement: “If a point lies on the bisector of an angle,

Bisecting the Vertical Angle of an Isosceles Triangle:
Construct an isosceles triangle ABC. Construct the bisector of the vertical angle. Examine the point where the angle bisector intersects the base of the triangle. Make a hypothesis about the angles created at the base of the triangle. Test your hypothesis for isosceles and non-isosceles triangles.



Draw an appropriate diagram in your notebook and complete the statement: “If the vertical angle of an

Angle Bisectors of a Triangle:
Construct a scalene triangle. Bisect two of the angles of the triangle. Select both bisectors and find the point of intersection by using the *Construct* menu. Bisect the third angle. All three of the angle bisectors will intersect at a point called the **incenter**.

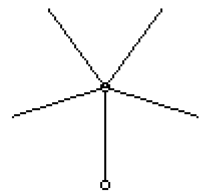


An **inscribed circle** is a circle drawn inside a triangle, touching each side of the triangle at only one spot. The **incenter** is the center of the inscribed circle. Use the intersection point of the angle bisectors to draw the inscribed circle using the circle tool. Repeat the construction to investigate whether an inscribed circle can be drawn in obtuse-angled, right-angled and isosceles triangles. Draw an appropriate diagram in your notebook and complete the statement: "If an incircle is to be constructed, then..."

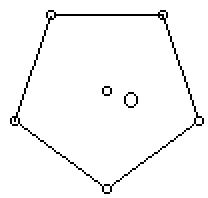
A landscape designer must determine the location of the largest possible circular pond that will fit into a triangular piece of lawn. Explain how the location can be determined. Use a diagram with your explanation.

Appendix: Investigating Polygons

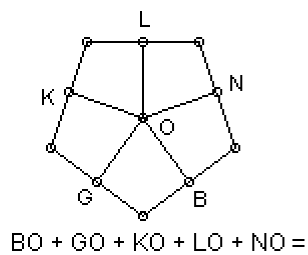
Construct a **pentagon** by rotating a line segment five times. What angle of rotation must be used?



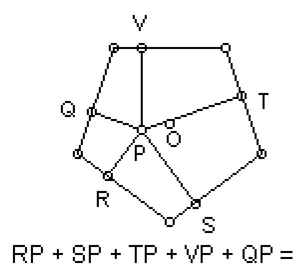
Place points at the tip of each line segment and at the center. Join the endpoints to make the pentagon, then hide the construction lines.



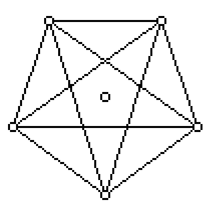
From the center, construct perpendicular lines to each side in order to measure the distance to each side. Record the distance. Calculate the sum of the distances from the center to the sides.



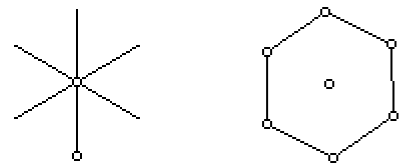
Pick another point P anywhere on the interior of the pentagon. Construct perpendiculars to each of the 5 sides. Measure the total distance from point P to all of the sides. Compare your answer to the previous question.



In a pentagon, draw line segments joining alternate vertices to form a star. What figure is formed by the inner part of the star? Repeat this process and examine the resulting pattern.

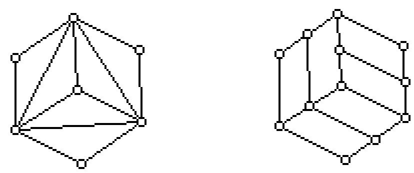


Investigating the Hexagon:
Construct a hexagon by rotating a line segment 6 times. What angle of rotation must be used? What type of triangles were formed in making the hexagon? (This might suggest another method to make a hexagon!) The hexagon has been divided into 6 congruent sections!



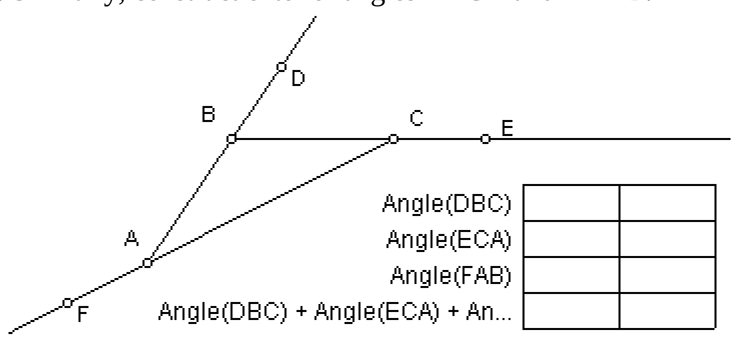
Investigate the hexagon to determine if it has the same characteristics discovered above for the pentagon. Write the results of your investigations in your journal.

The diagrams below illustrate other methods of dividing the hexagon into 6 congruent figures. Use angle theorems, midpoints, and parallel lines to draw line segments to divide the hexagon into 9 congruent figures, 12 congruent figures, and 24 congruent figures.



Exterior Angles of Polygons:
Plot a point A and draw a ray. Drag the point B along the ray AB and draw a ray from B. Select points C and A and use the *Construct* menu to construct ray CA.

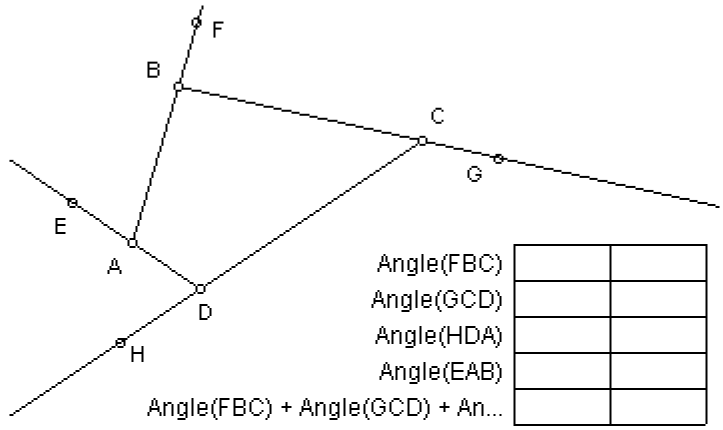
Select ray AB and use the *Construct* menu to construct point D on the ray, dragging the point to create exterior angle ∠DBC. Similarly, construct exterior angles ∠ECA and ∠FAB.



Measure each exterior angle. Calculate the sum of the angle measures using the *Measure, Calculate* menu.

Create a table to display the angle measures and sum.

Drag one of the external points to alter the angle measures. Add at least ten entries to the table, including all extreme and special cases. Record your results in your journal before going on.



Use the steps above to construct and explore the sum of the exterior angles of a quadrilateral.

Make a hypothesis about the exterior angles of a polygon.

Test your hypothesis by constructing and analysing a

a) pentagon
b) hexagon and
c) another polygon

Generalize the results, stating your conclusions in “If... then...” format.

Sum of the Interior Angles of a Polygon.

Design an investigation to determine the sum of the interior angles of a polygon. Use the steps *Explore, Form a Hypothesis, Test the Hypothesis, Generalize* and *Communicate the Result* to prepare a convincing argument Develop two different approaches and clear instructions for each approach, in order to explain your generalization to a fellow student.

Sum of the Interior and Exterior Angles of Regular Polygons

Design, carry out and report on an investigation to determine the sum of the *interior* angles of a regular polygon, and the sum of the *exterior* angles of a regular polygon.

Activity 3.7: Script It

Time: 225 minutes

Description

Students create scripts using dynamic geometry software to demonstrate and review geometric properties that they have discovered in previous activities.

Strand(s) and Expectations

Strand(s): Measurement and Geometry
Expectations: MG 3.02, .03, .04.

Planning Notes

There are two different methods for making a script in *The Geometer's Sketchpad*™. One way is to create a sketch and then make a script afterwards. Another method is to create the script as the sketch is being made. To start, it would probably be better to do the former so that students have completed a satisfactory sketch before they make a script. It is recommended that teachers work through this activity in advance to familiarize themselves with the process, if needed. It is relatively easy but the "bugs" should be worked out first.

Teaching/Learning Strategies

Student Activity: Script It

In this activity you are reviewing material that you learned in earlier activities while you create a script to record the commands used for your constructions.

1. Using *The Geometer's Sketchpad*™, construct a triangle. Then construct an inscribed circle.
2. You are now going to save the construction that you just made as a script. A script is a set of instructions that can be saved and replayed to demonstrate the construction process. *The Geometer's Sketchpad*™ writes the instructions for the construction you just made. Here is what you need to do:
   a) Use the point tool to point to all of the elements of your construction or, to make it easier, and to make sure that you don't forget anything, you can draw a rectangle around the entire sketch and all of the elements will be automatically selected.
   b) Click on *Work* in the upper tool bar. Highlight *Make Script.*
   c) You see a script appear on the right hand side of the screen.
   d) To play the script start another "Sketch". Start the sketch with three points and select those three points. Notice that the script lists three points as "Given". This means that those must be given initially before running the script.
   e) Now, click on *Play* in the Script window. Watch the construction begin.
   f) You can save your script (rename it if you want) so that it can be replayed again.
3. Now, create a script to construct a circumscribed circle

Teacher Facilitation: An advantage of creating scripts is that students can save the script and demonstrate the constructions for the teacher (or a peer), at any later time. This way, the teacher can assess a student's constructions by having the student play the script when the teacher circulates

Once students know how to create a script, they can go back over previous work and create scripts to illustrate various geometric properties that they have learned and test some of their conjectures. Teachers may want to select specific activities for students to revisit as needed.

Assessment/Evaluation Techniques

This final assessment activity should include the opportunity to use *The Geometer's Sketchpad* scripts to demonstrate fluency with geometric properties and relationships as well as the ability to explore, conjecture, and confirm or deny their conjectures.

Several ideas for assessment tasks, from which teachers can choose, are listed for referenced below:

1. Students use geometric constructions to complete a circle in "Oops! Glass Top" in the *Assessing Mathematical Understanding and Skills Effectively: Harvard Assessment Tasks* (see reference below). Assessment criteria and exemplars of written (rather than software) solutions are provided in "Glass Top", *Advanced High School Assessment Package I: Balanced Assessment for the Mathematics Curriculum* (see Resources below).
2. Students use angle properties to write a set of instructions for hitting one billiard ball to rebound to hit another given certain criteria and a diagram of a billiard table with two billiard balls, A and B, located on opposite sides of the table.
   "Bouncing Off The Walls", Harvard Project: Using the diagram of the billiard table, write a set of instructions (script) for:
   - Hitting ball A so that it bounces exactly once off the north or south wall before hitting ball B.
   - Hitting ball A so that it bounces exactly once off both the north and south walls before hitting ball B.
   - Hitting ball A so that it bounces exactly once off the north or south wall and exactly once off the east or west wall before hitting ball B.
3. Circle geometry can be assessed through the task of "Circling Trains", *Harvard Balanced Assessment in Mathematics Project* (see Resources). In this activity, students design the placement of amusement park attractions and pathways by placing attractions on circles and creating pathways between the attractions. They then continue these constructions given certain criteria.
4. "Mirror, Mirror II", *Harvard Balanced Assessment in Mathematics Project*, requires students to use angle properties to determine the placement of a mirror so that spotlights can illuminate a statue even though there is a wall between the light and the statue.

These activities can be assessed for Knowledge/Understanding of the geometric properties, Thinking/Inquiry as teachers observe students solving the problems, for Application as students apply their understanding of geometric properties to new contexts, and for Communication as students explain or justify their solutions.

Resources

Berkeley, Harvard, Michigan State, and Shell Centre. *Advanced High School Assessment Package 1: Balanced Assessment for the Mathematics Curriculum.* White Plains, N.Y.: Dale Seymour Publications, 1999. (Assessment criteria and exemplars of written (rather than software) solutions are provided.)

Harvard Balanced Assessment in Mathematics Project. *Assessing Mathematical Understanding and Skills Effectively*. 1996.