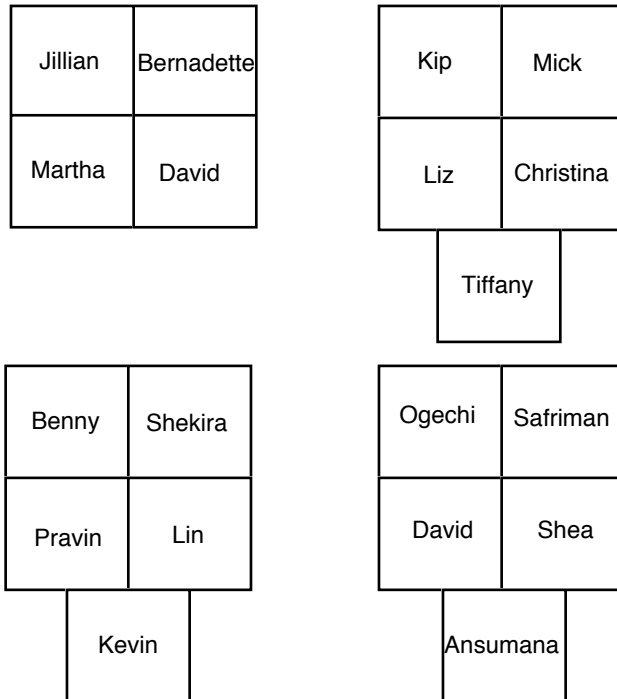


Seating Arrangement



1:05:31

Ball: More comments about the meeting? I'd really like to hear from as many people as possible what comments you had or reactions you had to being in that meeting yesterday. Shea?

Shea: Um, I don't have anything about the meeting yesterday, but I was just thinking about six, that it's a . . . I'm just thinking. I'm just thinking it can be an odd number, too, 'cause there could be two, four, six, and two, three twos, that'd make six...

Ball: Uh-huh . . .

Shea: And two *threes*, that it could be an odd and an *even* number. Both! *Three* things to make it and there could be *two* things to make it.

Ball: And the two things that you put together to make it were odd, right? Three and three are each *odd*?

Shea: Uh huh, and the other, the twos were even.

Ball: So you're kind of--I think Benny said then that he wasn't talking about *every* even number, right, Benny? Were you saying that? Some of the even numbers, like six, are made up of two odds, like you just suggested.

Other people's comments?

1:06:33

January 19, 1990. On this day, Ball began the class by asking students for comments on the meeting they had had the previous day where they talked about even and odd numbers with students from another class. A few minutes into the discussion, a boy named Benny made the observation that even numbers can be "made" from two other even numbers--like 4+4 or 6+6. This video segment opens with Ball asking if anyone has other comments. She calls on Shea who has his hand raised. Shea doesn't have a comment about the meeting but he has noticed something special about the number six which he claims can be an odd and even number.

Ball had assumed this clarified things. She had connected Shea's comment with what Benny had said a few minutes earlier when he pointed out that two even numbers combine to make other even numbers. Specifically, she thought Shea's point was that two odd numbers could also make an even number. She assumed they could now move on with their discussion of the meeting and then move from there onto the day's agenda. The class, however, picked up on and pursued Shea's point:

1:06:36

Ball: Tina?

Tina: I *disagree* with Shea when he says that six can be an odd number. I think six can't be an odd number because . . . look--*(she gets up and comes up to the board)*

Ball: *(interrupting)* Jillian, Bernadette?

Tina: Six can't be an odd number because this is *(she points to the number line, starting with zero)* even, odd, even, odd, even, odd, even, how can it be an odd number because *(starting with zero again)* that's odd, even, odd, even, odd, even, odd. Because zero's not a odd *number*.

Shea: Because six, because there can be three of something to make six, and three of something is like *odd*, like see, um, you can make two, four, six . Three twos to make that and two threes make it .

Kevin: But that doesn't--

Ball: Kevin?

Kevin: That doesn't necessarily mean that six is *odd*.

Students: Yeah.

Ball: Why not, Kevin?

Kevin: Just because two odd numbers add up to an even number doesn't mean it has to be odd.

Ball: What's the definition--Shea?-- what's our working--

Shea: Two odd numbers make--

Ball: Shea? What's our working definition of an even number? Do you remember from the other day the working definition we're using? What is it?

Shea: It's, um, that *(pause)* . . . I forgot.

Ball: Could somebody help us out with this? Because we need in the group to have an idea that we're working with. What's the working definition we're using? *(pause)* Do other people know it besides Liz and Shekira? *(pause)* I think other people do. Marta, do you know what the definition is that we've been using for an even number?

1:08:16

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*At this point Ball thought Shea was just confused about the definition for even numbers. She thought if they just reviewed that, he would see six fit the definition and was therefore even. She assumed after this they would be able to get on with the discussion.*

*Within a couple of minutes, they had settled on a definition of even numbers. Jillian said:*

*If you have a number that you can split up evenly without having to split one in half, then it's an even number.*

*So Ball turned to Shea in order to make the connection and clarify things:*

*Ball: Can you do that with six, Shea? Can you split six in half without having to use halves?*

*Shea: Yeah.*

*Ball: So then it would fit our working definition, then it would be even. Okay?*

*There was a pause.*

*Shea: And it could be odd. Three twos could make it.*

Ball: *Okay. One of the points here is that if it fits the definition then we would call it even. If it fits our working definition, then we would call it even.*

Shea: *It fits the definition for odd, too.*

Ball began to see that the issue was more complicated than she had thought.

Ball: *What is the definition for odd? Maybe we need to talk about that?*

*Before this they had had an explicit definition for even numbers only. Ball had assumed this was sufficient but apparently it was not. The class turned to discuss a definition of odd numbers and agreed that odd numbers were numbers you could not split up fairly into two groups. But this still did not satisfy Shea. He persisted with the observation he had made about what made six special:*

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1:11:24

Shea: You could split six fairly, *and* you can split six not fairly. You can like cut six in half, um .... there's like, say there's two of you and you had, and you had, um, six cookies and you didn't want to split it in *half* and so that each person would get three and you wanted to split it by twos. Each person would get um, two and there would be two left.

Ball: For which number now? For six?

Shea: Uh huh.

Ball: So, are you saying *all* numbers are odd then?

Shea: No, I'm not saying all numbers are odd, but . . .

Ball: Which numbers are *not* odd then?

Shea: Um. . . Two, four, six . . . um, six can be odd or even . . . eight

Students: *No. . . !*

Kip: I don't know how. Show us.

Shea: *Because* there's three twos. One, two. Three, four. Five, six.

Kip: Prove it to us that it can be odd. *Prove* it to us.

Shea: Okay. *(He rises and comes up to the board.)*

Ball: Does everybody understand what Shea's trying to argue? He's saying six could be even or it could be odd.

Students: I disagree . . . I don't think so . . .

Ball: Well, watch what he's going to prove and then you can ask him a question about it.

Shea: Well, see, there's two, *(he draws)* number two over here, put that there. Put this here. There's two, two, and two. And that would make six.



Kip: I know, which is *even!*

Lin: *I* think I know what he's saying.

Kip: Which is *even*, Shea.

Ball: Lin? *(to Shea)* Could you stay there? People have some questions for you.

Lin: I think what he is saying is that it's almost, see, I think what he's saying is that you have three groups of two. And three is a odd number so six can be an odd number *and* a even number.

Ball: Do other people agree with that? *Is* that what you're saying Shea?

Shea: Yeah.

Then it won't make sense that all numbers should be odd and even, because if all numbers were odd *and* even, we wouldn't be even having this *discussion!*

Ball: Okay, do other people agree with him? *(pause)* Lin, you disagree with that?

1:15:40

Lin: Yeah, I disagree with that because it's not according to like . . . here, can I show it on the board?

Ball: Um hm.

Lin: *(She comes up to the board.)* It's not according to like. . .

*Ball decided that if they were going to pursue this, more people should be invited to join in. So she turned to the rest of the class.*

Ball: Rania, can you watch what Lin's doing?

1:17:21

Lin: . . . how many groups it is. Let's say that I have *(pauses)* Let's see. If you call six an odd number, why don't *(pause)* let's see *(pause)* let's see--ten. One, two . . . *(draws circles on board)* and here are ten circles. And then you would split them, let's say I wanted to split, spit them, split them by twos. . . One, two, three, four, five . . . *(she draws)*



then why do you not call *ten* a, like-

Ball: Are people following this disagreement? This is an important thing that I didn't even realize we were disagreeing about, so it's important to see if we can try to figure this one out. What are you going to show?

Ogechi: Well, I don't really need to show something.

Ball: You *don't* need to show something? What are you trying to say?

Ogechi: I just wanted to say if, um, you wanted an odd number, you just have to take one off of it.

Shea: I disagree with myself.

Ball: Why would that work?

Lin: . . . a, an odd number and an even number, or why don't you call *other* numbers an odd number and an even number?

Ogechi: Um, because usually odd numbers are. . . like, like. . . *(picks up chalk)*

Shea: I didn't think of it that way. Thank you for bringing it up, so --I say it's--ten can be an odd and an even.

Ball: *(interrupting)* Tina, can you hear Ogechi okay?

Lin: Yeah, but what about ...

Ogechi: this, kind of like have ones in the middle? Like five *(she draws):*



Liz: Ohh!!!

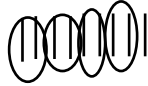
and there's a one in the middle, or, um nine: *(she draws)*

Lin: What about *other* numbers?! Like, if you keep on going on like that and you say that other numbers are odd and even, maybe we'll end it up with *all* numbers are odd and even.



Lin: I think it's better with circles.

Ogechi: *(she circles groups of two in her drawing of nine hash marks)*  
This two together, this two together . . . this two together,  
and this two together.



There's one left. And even numbers--like six--*(she draws)*



--you can't get *anything* in the middle. There isn't one left.

Ball: So you're saying the even numbers are the ones where you can group them all by two's and the odd ones are the ones where you end up with one left over?

Lin: I think I agree.

Shea: But, if six is an even number, then how come there's three here and there's not one left out.

Ogechi: *Because*, even numbers are like things *like* this, even numbers have two in them, and also odd numbers have two in them, except they have one left.

Lin: Yeah.

Ball: Okay, so, Ogechi, you actually are suggesting a definition, I think. Let's have everybody hear that one more time.

Shea: We're not *using* nine!

Ball: Listen carefully, Ogechi's proposing a definition for odd numbers.

Shea: But we're not using nine.

Ball: Can you listen to her one more time? Say again one more time what you're saying the definition is of an odd number.

Ogechi: Well, an odd number is something that has one number left over.

Ball: After you do what?

Ogechi: After you circle the two's.

Ball: Any questions? Are you clear about what she's trying out?

Shea: It doesn't always have to--

Ball: Just a second, Shea. Let's make sure people understand what she's suggesting. Who thinks they could come up and try a number on the board using her definition, to test to see if it's even or odd. That would be a way of seeing if we understand what she's trying to say. *(pause)* Bernadette, you think you could try one?

Bernadette: Yup.

Ball: Ogechi, watch and see if she's using the way you're thinking about it. What number are you going to experiment with?

Bernadette: Um, I'll experiment with twenty . . .

Ball: What would you predict . . .

Bernadette: . . . twenty-one.

Ball: . . . what would you think twenty-one should be?

Bernadette: Odd.

Ball: Okay, so if her method works, what will happen.

Bernadette: There would be one left over.

Ball: Okay, everyone watch now and see if Bernadette's experiment works out. *(Writes Ogechi's proposal on the board.)*

1:20:33

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*Ball decided it would be useful to really get "inside" of what Ogechi was saying. Her intent was simply to have the rest of the class consider the usefulness of what Ogechi had proposed and then to be able to come back and settle, finally, this issue about six being odd.*

*The class was becoming increasingly invested in what Shea was arguing.*

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1:27:10

Ball: Okay. Ogechi, what are you shaking your head about?

Ogechi: I don't agree that six *can* be odd.

Ball: Can you show us why?

Ogechi: I already did both of them. Well, my conjecture was that if it's even number, you can have all the twos circled and I already *have* all the twos circled. *(motions to her drawing on the board)*



Ball: Tina?

Tina: I agree with her. And I have a question for Shea. If six is odd, then what is five?

Shea: I'm not saying that it's always odd, I'm saying that it can be odd and even.

Tina: It can't be *both*.

Shea: It *can*.

Tina: No, it can't.

Ball: Why can't it be both, Tina?

Tina: Hmm?

Ball: Why can't it be both?

Tina: Because right here you can't go on the number line, you can't do this. *(Points to the number line, starting with 1)* Even, odd, even, odd, even, odd, even, odd, you can't do that.

Shea: I'm not saying all of those are even and odd, I'm not saying they're all even or odd . . . I'm just saying six.

Tina: How can six be both?

Shea: Because! See, there's like three um, there's three twos and three is an odd number, so there's three twos and that's an odd, so that would probably make six and you can circle all of them, and make three on each side and that's, um, an odd number.

Ball: Rania?

Rania: I disagree.

Ball: Go ahead.

Rania: Can I show my way on the board? *(Rania goes up and begins drawing.)*

Ball: Um hm. Tina, you can sit down. Lin, you can sit down. Thanks. Ogechi, I'll just have you stand there for a minute since this is the definition you were suggesting, okay? Rania, what are you disagreeing with? Rania? What are you disagreeing with?

Rania: With Shea.

Ball: Okay, and what are you going to show us?

Rania: I'm going to show . . . see, it doesn't . . . um, it doesn't matter how much circles there are, it matters when you like put one, two, three, four, five, six . . . it doesn't matter about, it doesn't mean . . . how much times you circle two, it doesn't *prove* that six is an odd number.

Ball: Other people's reactions? (*bends over to speak to Liz and Kip about participating in the discussion*) Comments to what Rania said? She's saying it doesn't matter how many circles you make when you group by twos, what matters is if you *can* group by twos. Kevin?

Kevin: I agree.

Ball: You agree with Rania?

Kevin: Yeah.

Ball: Ogechi?

Ogechi: I agree. That's kind of like what I was going to say to him. Um, what Shea said doesn't have to do with my conjecture.

Ball: Okay, so you're disagreeing that it's the number of groups. And Shea, are you saying that the number of groups matters?

Shea: Yeah, because there's like . . .

Ball: Three groups of two, right? So what about ten? Ten has five groups of two.

Shea: Yeah, so that would be odd.

Ball: I thought you just said six was the only one?

Shea: Well . . . six and ten. Because there's five groups of two in ten and there's three groups of two in six.

Ball: Anybody want to comment on this, or is this one of the examples of something we've got to let rest a little bit now? Does anybody else want to comment on what Shea's saying? Or on what Rania, Ogechi, Tina and so forth have said?

1:31:12

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*Ball tried one more time to close the discussion. It was her sense they had spent too much time on this— that they should really move on and come back to it later. She could let Shea rest, and the class could return to this issue another day. Knowing Shea, and knowing how tenacious he could be at times, Ball was worried about him being backed into a corner, which might be hard to come out of during this particular class period.*

*But, once again, the discussion picked up, spurred on by the students. Kip badly wanted to make a point and then the conversation was off and running again. The children talked about 6 and 10, trying to think about Shea's claim that these numbers could be both even and odd.*

*Ball decided she could play more of a role in pushing this. She did what Lin had done earlier— show another number like 6 that, following Shea's reasoning, would be considered both even and odd. Ball proposed fourteen. She was thinking that this might help him see there were many even numbers with this property—having odd numbers of groups of two. He just agreed this was another case. Ball was curious about why he was interested in this, so she asked him why this was a helpful thing to notice.*

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1:35:53

Ball: Okay, I guess the problem that I'm having, but then I think maybe we're going to have to stop with this, is that if you have a definition that counts the number of groups and nobody else is counting the number of groups, that's a little bit confusing. Why do you think it's important to keep track of the number of groups, Shea?

Shea: Because that would make it an odd number.

Ball: And what's the advantage? Why is that *useful* if we call some numbers "odd-and-even," how does that help us?

Shea: Um . . .

Ball: We want these definitions to help us talk to each other, right? To help us make us understand each other and why...

Shea: I'm not *sure* it can *help* us.

Ball: Why would you like us to agree with you? Why would that be helpful if we all agreed that it was both odd and even? Can you convince us that that would be helpful?

Shea: It's just helpful to know it can be odd and even, because I'm just saying it *can*, I'm didn't, I just, just saying that it's . . . I just *say* that, I'm not . . .

1:36:56

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*His answer caught Ball off guard. To Shea, this was not useful or helpful--it was just interesting. She was intrigued; she was herself, convinced that mathematics has aesthetic qualities, that everything we do in school need not be "practical" or related to the "real world."*

*Ball was thinking, again, that they really should move on now. This discussion had taken up almost half an hour of the one-hour math class. And she thought they had made some progress on their definitions of even and odd numbers.*

*But Rania had her hand in the air and was waving for attention.*

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1:36:56

Ball: Why don't we stop with this now, Rania? I think that um, we've discussed this from a number of angles. I think Shea has. . .

Rania: *(interrupting)* Twenty doesn't work.

Ball: Twenty doesn't work in what sense?

Rania: There's ten groups.

Ball: Uh huh, and ...

Rania: Ten's not an even, I mean a odd number. It doesn't work.

Ball: I'm confused.

Rania: He said, if there's um, um, if there's an odd number--I wrote twenty, two, four, six . . .

Ball: Shea, you're going to have to help because she's talking to you and I'm not sure what she's trying to show.

Shea: I'm not *saying* that twenty can be an odd number. I'm saying *twenty-two* can be an odd number--

Tina: Mmm-hmm.

Shea: Because, yes, it can. Look.

Rania: How come twenty-four can't be one either?

Shea: Yes, it can.

Rania: No it can't. One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve. Twelve's an even number.

Shea: See, like, um, here's twenty-two. *(begins to draw 22 lines)*

Rania: Here you are. *(pointing to her drawing below)* Here's twenty-two.

Shea: Okay. Here's twenty-two. . . *(circles and counts the eleven groups of two for Rania)*

Ball: Is there a pattern to the ones that Shea is calling even and odd? Is there a pattern?

Rania: *(and several other students)* Yeah.

- Shea: Uh huh. Yeah. I know what it is.
- Ball: Shea, what's the pattern?
- Shea: Every four numbers, like um, there's one starting out like that, and it can be an odd number, and then four more . . .
- Ball: Why?
- Shea: The fourth one would be another odd number.
- Ball: Why?
- Shea: Because you can split it in odd groups. Odd number of groups.
- Ball: Rania?
- Rania: (and several other students) See, I think this is ... um, you keep on going . . . each one is--one (*is making marks on the board*). . . so two is one . . . so odd numbers. . . so, see, this is what I think this is the pattern. Two can be a odd number and even--this is what Shea, I think Shea is ...
- Ball: I don't know if he proposed that 2 is one of them though.
- Shea: Two isn't, two isn't, because you can't make it an odd number. There's only one group there.
- Rania: (*and several other students*) One's an odd!!
- Rania: (*repeating*) One's an odd!
- Shea: So two can . . .
- Ball: Okay, so . . .
- Rania: So four can't be...because that's *two* groups. Six can. (*points at even numbers on the number line, starting with eight*) This can. Can't. Can. Can't.
- Ball: Is that right, Shea?
- Shea: Yeah.
- Ball: Ogechi? (*realizes that she is interrupting Rania*) Oh, sorry.
- Rania: So some odd . . . some even numbers, this is what . . . I think Shea's saying some even numbers in a pattern can be um, can be even *and* odd and some can't.
- Shea: Yeah, that's what I'm trying to say.
- Ball: Ogechi?
- Ogechi: Well, I just think, um, that just because you put twenty-two there and that makes eleven, that that doesn't mean that it's an odd number. My conjecture, I think it's always true, because if all twos are circled in a number that's even, then it's an even number.

1:40:30

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*The class talked for a few more minutes. Ball asked the children to write down what they were thinking at this point about what the definitions were for even and odd numbers. She asked them to give an example of each. What Ball found was that all the students who wrote--and about five did not write anything--were articulating pretty conventional ideas about even and odd numbers and they were able to articulate definitions, not just give examples.*

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**Postscript:** The next chapter of this story can be found in Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal*, 93(4), 373-397.

NAME	GENDER	RACE	COUNTRY	ENGLISH PROFICIENCY	HOW LONG AT THIS SCHOOL <sup>1</sup>
Ansumana	M	African American	U.S.A.	native speaker	just started
Benny	M	White	Ethiopia	fluent	3 years
Bernadette	F	White	Canada	native speaker	4 months
Christina (Tina)	F	African American	U.S.A.	native speaker	12 months
David	M	Asian	Indonesia	developing	3 years
Jillian	F	White	U.S.A.	native speaker	3 years
Kevin	M	African American	U.S.A.	native speaker	just started
Kip	M	African Black	Kenya	fluent	3 years
Lin	F	Asian	Taiwan	fluent	2 years
Liz	F	White	U.S.A.	native speaker	3 years
Marta	F	Latina	Nicaragua	beginning	4 months
Mick	M	White	U.S.A.	native speaker	2 years
Ogechi	F	African Black	Nigeria	fair	3 years
Pravin	M	White	Nepal	beginning	5 months
Rania	F	White	Egypt	good	3 years
Safriman	M	Asian	Indonesia	developing	12 months
Shea	M	White	U.S.A.	native speaker	2 years
Shekira	F	African American	U.S.A.	native speaker	4 months
Tiffany	F	White	U.S.A.	native speaker	just started

<sup>1</sup>NOTE: This column reflects the length of time the child had been in this school as of 10/89. No one had been in this class longer than a month (since the beginning of school).